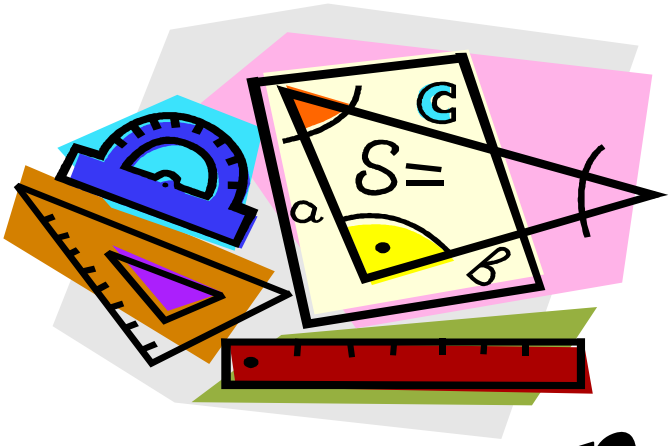
Level III



Do NOT open until you are told to do so.

March 21, 2019

1. Zantac can relieve acid reflux. The recommended dosage for a child is 5 mg/kg/day. Zantac comes in liquid form where the concentration of the medicine is 15 mg per mL. If a child with acid reflux weighs 38.5 pounds, how many milliliters of Zantac should be taken each day? Assume 1 kg = 2.2 lb.

   a. $6\frac{2}{3}$ mL b. 6 mL c. 1312.5 mL d. $5\frac{5}{6}$ mL e. 55 mL

2. A book is to have 250 pages that will be numbered with Arabic numerals beginning with 1. How many times will the digit 2 be used in numbering the pages?

   a. 107 b. 106 c. 105 d. 56 e. 85

3. The solution of $\begin{cases} 3x + 4y > 12 \\ 5x - 6y \geq -30 \end{cases}$ intersects more than one quadrant. Which quadrant does NOT include some part of the solution set of $\begin{cases} 3x + 4y > 12 \\ 5x - 6y \geq -30 \end{cases}$?

   a. I b. II c. III d. IV e. all quadrants are included

4. The coordinates of ΔABC are $A(0,0)$; $B(5,0)$; and $C(0,10)$. Point A is reflected over $\overleftrightarrow{BC}$ and labeled A'. What are the coordinates of A'?

   a. $(4,10)$ b. $(10,5)$ c. $(10,4)$ d. $(4,8)$ e. $(8,4)$

5. Sue works weekdays for \$10 an hour, Saturdays for \$15 an hour, and Sundays for \$20 an hour. If she worked 180 hours last month and earned \$2315, how many more weekday hours than Sunday hours did she work last month?

   a. 75 b. 77 c. 80 d. 82 e. 85

6. How many solutions are there to the equation $|2x^2 - x - 1| = x$?

   a. 2 b. 0 c. 5 d. 4 e. 1

7. For real numbers x and y, define the binary operation # by: $x \# y = \dfrac{xy^2 + yx^2}{5 + x^2y^2}$. Calculate 2 # (5 # 2).

   a. $\dfrac{17}{105}$ b. $\dfrac{17}{61}$ c. $\dfrac{32}{61}$ d. $\dfrac{16}{31}$ e. $\dfrac{61}{17}$

8. The roots of $ax^2 + bx + c = 0$, where a, b, and c are real numbers with a nonzero, are r and s. If $\dfrac{r}{1+r}$ and $\dfrac{s}{1+s}$ are the roots of $x^2 + dx + f = 0$ (d, f are real), find $d + f$.

   a. $\dfrac{b-c}{a-b+c}$ b. $\dfrac{b-c}{b-a+2c}$ c. $\dfrac{3c-b}{a-b+c}$ d. $\dfrac{3c-b}{b-a+2c}$ e. $\dfrac{b+c}{b-a+2c}$

9. A standard six-sided die is biased so that $P(n) = nP(1)$ for all n such that $1 \le n \le 6$. What is the probability of rolling a sum of 8 in three successive rolls of this die?

   a. $\dfrac{1}{49}$ b. $\dfrac{4}{147}$ c. $\dfrac{5}{147}$ d. $\dfrac{2}{49}$ e. $\dfrac{1}{21}$

10. How many solutions (a,b) where a and b are integers are there for the equation $a^4 + 2a^2b^2 + b^4 - 6a^2 - 6b^2 - 27 = 0$?

    a. 2 b. 4 c. 6 d. 8 e. 10

11. Assume there is a 90% chance that a person chosen at random is right-handed. What is the probability that exactly two out of three people chosen at random will be right-handed?

a. 0.972 b. 0.729 c. 0.271 d. 0.243 e. 0.81

12. Define a function by $f(x)=\begin{cases} |x| & if \quad x\le 0 \\ f(x-2) & if \quad x>0 \end{cases}$. Calculate $f(2.75)-f(3.25)$.

a. 2 b. − 1.5 c. 0.5 d. − 0.5 e. 2.5

13. Which of the following is equivalent to $\left(\sqrt{2}+\sqrt{2}i\right)^{2020}$?

a. -4^{2020} b. $-2^{2020}i$ c. 2^{2020} d. $-8^{505}i$ e. -4^{1010}

14. How many positive integers k are there with the property that $k!$ is not divisible by 75?

a. 19 b. 18 c. 4 d. 9 e. 11

15. Two fair, ten-sided dice are rolled. What is the probability that the sum of the two numbers is prime?

a. $\frac{39}{100}$ b. $\frac{9}{25}$ c. $\frac{37}{100}$ d. $\frac{7}{20}$ e. $\frac{17}{50}$

16. Which interval is a subset of the domain of $f(x)=\sqrt{\dfrac{x-\dfrac{3}{x-4}}{\dfrac{x+2}{x-2}}}$?

a. $(-2,-0.5)$ b. $(2,5)$ c. $(4.5,\infty)$ d. $(-\infty,-2)$ e. none of these

17. Let $a=2^{1/3}$, $b=3^{1/5}$, and $c=10^{1/10}$. Which of the following is a true statement?

a. $a<b<c$ b. $a<c<b$ c. $c<b<a$ d. $b<c<a$ e. $b<a<c$

18. There exists positive integers A, B, and C, with no common factor greater than 1, such that $A\log_{200}5+B\log_{200}2=C$. What is $A+B+C$?

a. 6 b. 7 c. 8 d. 9 e. 10

19. How many integers *n* satisfy $n^4+5n<5n^3+n^2$?

a. 4 b. 1 c. 3 d. 5 e. 2

20. Morse code involves transmitting dots "•" and dashes "▬". An agent attempted to send a five-character code five different times, but only one of the five transmissions was correct. However, it is known that each erroneous transmission had a different number of errors than the others, and no transmission had five errors. The five transmissions sent are shown below, which is the correct one?

a. • ▬ ▬ ▬ • b. ▬ ▬ • • ▬ c. • ▬ • ▬ • d. • • • • • e. • ▬ ▬ • ▬

SHORT ANSWER
Place the answer in the appropriate space.

66. Recall that $i = \sqrt{-1}$. For how many integers n is $(n+i)^4$ an integer?

67. What is the sum of the digits of $\dfrac{(10^{99}+10^{98}+10^{97}+\ldots+10^{2}+10^{1}+10^{0})}{11}$?

68. There are 100 women over the age of twenty in a certain town. 95 have a cell phone, 65 are married, 90 own a car, and 55 have children. What is the least possible number of these 100 women who are married, have a cell phone, own a car, and have children?

69. What is the remainder when 1! + 2! + 3! + … + 100! is divided by 18?

70. Every so often, a peculiar professor buys *n* snacks at the store and then arranges them in a circle. He eats one snack each day and gives the last one remaining to his dog. He begins at the top of the circle (#1), and then, moving clockwise, eats *every other* snack remaining on the table. For example, if he buys 5 snacks, he eats #1, skips #2, eats #3, skips #4, eats #5, skips #2, eats #4, then gives #2 to his dog. If he buys 75 snacks, which one will his dog eat?

1. D
2. B
3. C
4. E
5. B
6. A
7. C
8. A
9. B
10. B
11. D
12. C
13. E
14. D
15. C
16. E
17. D
18. A
19. C
20. A

66. 3
67. 50
68. 5
69. 9
70. 22
71.