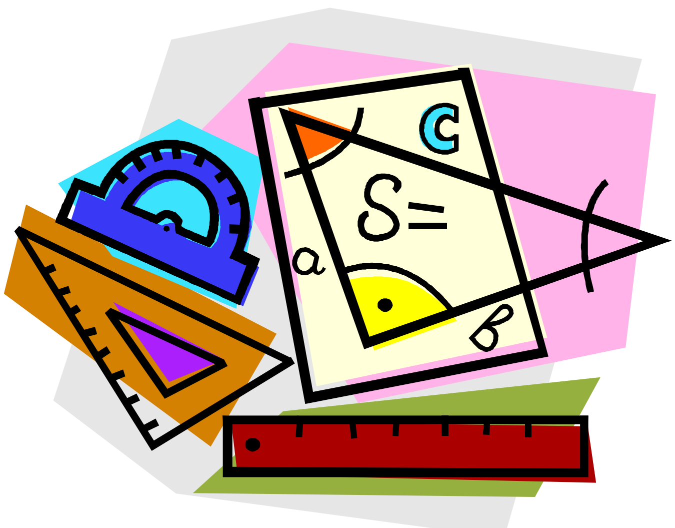
Comprehensive



Do NOT open until you are told to do so.

March 21, 2019

1. Zantac can relieve acid reflux. The recommended dosage for a child is 5 mg/kg/day. Zantac comes in liquid form where the concentration of the medicine is 15 mg per mL. If a child with acid reflux weighs 38.5 pounds, how many milliliters of Zantac should be taken each day? Assume 1 kg = 2.2 lb.

   a. $6\frac{2}{3}$ mL b. $5\frac{5}{6}$ mL c. 1312.5 mL d. 6 mL e. 55 mL

2. Which interval is a subset of the domain of $f(x)=\sqrt{\dfrac{x-\frac{3}{x-4}}{\frac{x+2}{x-2}}}$?

   a. $(-2,-0.5)$ b. $(2,5)$ c. $(4.5,\infty)$ d. $(-\infty,-2)$ e. none of these

3. The coordinates of ΔABC are $A(0,0)$; $B(5,0)$; and $C(0,10)$. Point A is reflected over $\overleftrightarrow{BC}$ and labeled A'. What are the coordinates of A'?

   a. $(4,10)$ b. $(10,5)$ c. $(10,4)$ d. $(8,4)$ e. $(4,8)$

4. For real numbers x and y, define the binary operation # by: $x\#y=\dfrac{xy^2+yx^2}{5+x^2y^2}$. Calculate 2 # (5 # 2).

   a. $\dfrac{17}{105}$ b. $\dfrac{17}{61}$ c. $\dfrac{32}{61}$ d. $\dfrac{16}{31}$ e. $\dfrac{61}{17}$

5. The solutions of $ax^2+bx+c=0$, where a, b, and c are real numbers with a nonzero, are r and s. If $\dfrac{r}{1+r}$ and $\dfrac{s}{1+s}$ are the roots of $x^2+dx+f=0$ (d, f are real), find $d+f$.

   a. $\dfrac{b-c}{a-b+c}$ b. $\dfrac{b-c}{b-a+2c}$ c. $\dfrac{3c-b}{a-b+c}$ d. $\dfrac{3c-b}{b-a+2c}$ e. $\dfrac{b+c}{b-a+2c}$

6. The positive integers a, b, and k satisfy the equation $\frac{a}{b}=\frac{a-b}{k}$. How many solutions are there for which $b \le 8$?

a. 10 b. 11 c. 12 d. 13 e. 14

7. A unit square in the xy-plane with opposite vertices at $(0,0)$ and $(1,1)$ is rotated clockwise around the point $(1,0)$ until the vertex at $(1,1)$ is at $(2,0)$. What is the area of the region traced out by this rotation?

a. $\frac{\pi}{2}+1$ b. $\pi-1$ c. $\frac{\pi}{4}+2$ d. $\frac{3\pi}{2}-2$ e. $2\pi-4$

8. Scott rolls a red die. His score is equal to the result of the roll. Then Wendy rolls a white die. Her score is the larger of the red die and the white die results. Finally, Deb rolls a blue die. Her score is the largest of the red, the white, and the blue die results. The dice are fair and six-sided. Let a = Deb's score, b = Wendy's score, and c = Scott's score. Obviously, $a \ge b \ge c$. What is the probability that $a > b > c$?

a. $\frac{1}{3}$ b. $\frac{5}{27}$ c. $\frac{1}{6}$ d. $\frac{2}{27}$ e. $\frac{5}{54}$

9. What is the sum of all solutions in the interval $[0,\pi]$ of the equation $\sin(8x)\cos(2x)=\sin(2x)\cos(8x)$?

a. 3π b. $\frac{10\pi}{3}$ c. $\frac{11\pi}{3}$ d. $\frac{7\pi}{2}$ e. 4π

10. The solutions of the quadratic equation $x^2+ax+b=0$ are $x=\sin(15°)$ and $x=\cos(15°)$. What is the value of a^4-b^2?

a. 2 b. −2 c. $\frac{35}{16}$ d. $i-2$ e. 0

11. Let $\sin(t)=a$, $\cos(t)=b$, and $\tan(t)=c$. Which of the following is equivalent to $\sin(-t+4\pi)+3\cos\left(\frac{\pi}{2}-t\right)-\tan(t-3\pi)$?

a. $a-3b-c$ b. $-a+3b+c$ c. $-a+3b+\frac{c}{2}$ d. $2a-c$ e. $4a+c$

12. How many integers satisfy $n^4+5n<5n^3+n^2$?

a. 3 b. 1 c. 4 d. 5 e. 2

13. There exists positive integers A, B, and C, with no common factor greater than 1, such that $A\log_{200}5+B\log_{200}2=C$. What is $A+B+C$?

a. 6 b. 7 c. 8 d. 9 e. 10

14. Let $a=2^{1/3}$, $b=3^{1/5}$, and $c=10^{1/10}$. Which of the following is a true statement?

a. $a<b<c$ b. $a<c<b$ c. $c<b<a$ d. $b<a<c$ e. $b<c<a$

15. Which of the following angles satisfies $\cot(\theta)\sec(\theta)<0$?

a. $450°$ b. $\frac{27\pi}{5}$ c. $-\frac{\pi}{2}$ d. $\frac{4\pi}{7}$ e. $-189°$

16. The sides of a triangle are 14 in, 16 in, and 18 in. What is the length of the shortest altitude?

a. $\frac{8\sqrt{5}}{3}$ in b. $4\sqrt{5}$ in c. $8\sqrt{5}$ in d. $\frac{64\sqrt{5}}{9}$ in e. $\frac{16\sqrt{5}}{3}$ in

17. Two fair, ten-sided dice are rolled. What is the probability that the sum of the two numbers is prime?

a. $\frac{7}{20}$ b. $\frac{37}{100}$ c. $\frac{9}{25}$ d. $\frac{19}{50}$ e. $\frac{39}{100}$

18. What is the product of all solutions on the interval $[0,2\pi)$ of the equation $\sin^2(x)\sec(x)+2\sin^2(x)=\sec(x)+2$?

a. $\frac{2\pi^4}{3}$ b. $\frac{35\pi^4}{48}$ c. $\frac{8\pi^2}{9}$ d. $\frac{4\pi^3}{9}$ e. 0

19. Which describes the graph (in $\mathbb{R}^3$) of all solutions of the system $\begin{cases} 2x-6y-8z=15 \\ -8x-8y+6z=-65 \\ x-19y-17z=5 \end{cases}$?

a. a point b. a plane c. a line d. two lines e. two planes

20. Morse code involves transmitting dots "•" and dashes "▬". An agent attempted to send a five-character code five different times, but only one of the five transmissions was correct. However, it is known that each erroneous transmission had a different number of errors than the others, and no transmission had five errors. The five transmissions sent are shown below, which is the correct one?

a. • • • • • b. • ▬ ▬ ▬ • c. ▬ ▬ • • ▬ d. • ▬ • ▬ • e. • ▬ ▬ • ▬

SHORT ANSWER
Place the answer in the appropriate space.

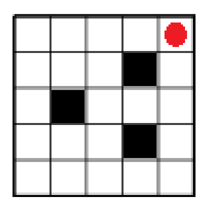
66. How many different prime numbers are factors of N if $\log_2\left(\log_3\left(\log_5\left(\log_7 N\right)\right)\right)=2$?

67. Four sets each have K number of elements. The intersection of any two sets has $\frac{K}{2}$ elements. The intersection of any three sets has $\frac{K}{3}$ elements. The intersection of all four sets has $\frac{K}{4}$ elements. If the union of all four sets contains 75 elements, what is the value of K?

68. Let S be a set of at least two consecutive integers whose sum is 99. For example one such set is $\{32,33,34\}$. How many distinct sets S exist?

69. A sequence of numbers has 6 as its first term, and every term after the first is defined by $a_{n+1}=\begin{cases}\frac{1}{2}a_n & a_n \text{ is even}\\ 3a_n+1 & a_n \text{ is odd}\end{cases}$. Thus the first four terms are 6,3,10,5. What is the 100th term?

70. A checker is placed on a 5×5 checkerboard as pictured. The checker may be moved one square at a time but only to the left or down. Also, the checker may not move to any of the three black squares. In how many different ways can the checker be moved to the lower left corner of the board?



1. B
2. E
3. D
4. C
5. A
6. D
7. A
8. E
9. D
10. C
11. D
12. A
13. A
14. E
15. B
16. E
17. B
18. C
19. C
20. B

66. 1
67. 36
68. 11
69. 4
70. 10