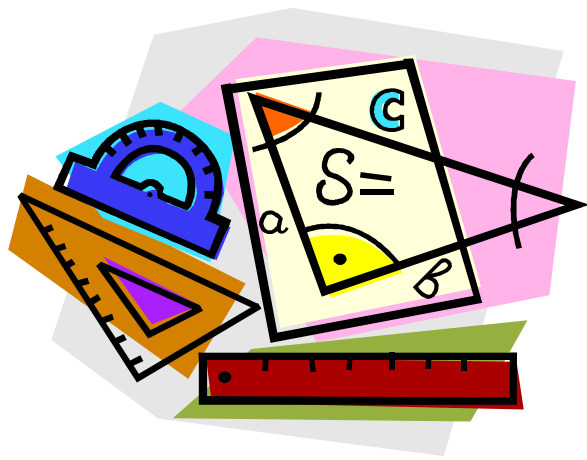


Comprehensive



**Do NOT open until
you are told to do so.**

- A drawer contains 16 socks, 6 of which are orange. When three socks are drawn (without replacement) at random, what is the probability that at least one sock is orange?
 - $\frac{3}{8}$
 - $\frac{5}{8}$
 - $\frac{3}{14}$
 - $\frac{11}{14}$
 - $\frac{17}{56}$
 - In triangle ABC , $AB = 2$, $BC = 1$. Suppose side AC and the median from vertex B to side AC have the same length. What is the value of $(AC)^2$?
 - $\frac{3}{2}$
 - 2
 - $\frac{9}{4}$
 - 3
 - none of these
 - Aziza runs a dragon fruit delivery service. She charges \$4 for each dragon fruit and a \$1 delivery fee. Aziza creates data set A from how many dragon fruit were ordered in each delivery, and data set B from how much she charged for each delivery. What is the ratio of the standard deviation of data set B to the standard deviation of data set A?
 - 1
 - 2
 - 4
 - 5
 - none of these
 - Suppose that for all real $x \notin \{-5, 1\}$, $f\left(\frac{2x-3}{x+5}\right) = \frac{3x+2}{x-1}$. Determine $f(x^2)$.
 - $\frac{9x^2+12x+4}{x^2-2x+1}$
 - $\frac{13x^2+13}{6x^2+1}$
 - $\frac{4x^2-12x+9}{x^2+10x+25}$
 - $\frac{3x^2+2}{x^2-1}$
 - $\frac{6x^2-5x-6}{x^2+4x-5}$
 - Siobhan and Nguyen have identical bags of marbles. They each contain five marbles – one pink, one purple, one brown, one gray, and one tan. Siobhan randomly selects a marble from Nguyen's bag and puts it in her bag. Then, Nguyen randomly selects a marble from Siobhan's bag and puts it in his bag. What is the probability that, after this process, the contents of the bags are still identical?
 - $\frac{1}{10}$
 - $\frac{1}{6}$
 - $\frac{1}{5}$
 - $\frac{1}{3}$
 - $\frac{1}{2}$
 - The ratio of the interior angles of two regular polygons with sides of length 1 unit is 5:2. What is the sum of the perimeters of the polygons?
 - 7
 - 12
 - 14
 - 15
 - 20
 - Right triangle ABC is such that angle B is a right angle and $BC = 60$. Let point D be on segment AB such that $AD = 1$. Segment BD is the diameter of a semicircle that is tangent to segment AC at point E . Determine the area of the semicircle.
 - 30π
 - 36π
 - 60π
 - 72π
 - 144π
 - Which of the following is a simplification of $(\log_4 9)(\log_3 7)(\log_7 8)$?
 - 3
 - 6
 - 12
 - $\log_{14} 24$
 - $\log_{84} 504$
 - Suppose the arithmetic mean of x^2 and y^2 equals $3x + y + 45$. What is the largest possible value of $3x + 4y$?
 - 53
 - 63
 - 70
 - 73
 - 81
-

10. Twenty-two points are equally spaced on a circle. From these points a certain amount will be chosen at random. What is the minimum number of points that must be selected to guarantee that four vertices of at least one rectangle are chosen?
- a. 5 b. 11 c. 13 d. 15 e. 17
11. Let a, b and x be real numbers such that:
 $\log_{a-b}(a+b) = 3^{a-b}$, $\log_{a+b}(a-b) = 8 \cdot 6^{b-a}$, $a^2 - b^2 = 3^x$
 Determine the value of x .
- a. 16 b. 18 c. 24 d. 27 e. 28
12. Suppose $T(t) = 125e^{-kt} + 75^\circ\text{F}$ is the temperature of a pot of coffee t minutes after it is brewed. If the pot's lid is on, $k = 0.02$, and if the pot's lid is off $k = 0.1$. Let t_{on} and t_{off} be the different times required for the pot to cool to 150°F with the lid on and off, respectively. What is $\frac{t_{\text{on}}}{t_{\text{off}}}$?
- a. $\frac{1}{20}$ b. $\frac{1}{5}$ c. $\frac{5}{3}$ d. 2 e. 5
13. The sides of a triangle have lengths 12, 16, and x , where x is an integer. For how many values of x is the triangle acute?
- a. 9 b. 10 c. 15 d. 16 e. 23
14. Suppose a three-digit number is chosen at random and the number of distinct digits is counted (for example, the number 522 has 2 distinct digits). What is the expected number for distinct digits?
- a. 2.439 b. 2.475 c. 2.5 d. 2.71 e. 2.75
15. Calculate
- $$\sum_{k=1}^{59} \ln(\tan(k^\circ)) + \sum_{k=61}^{89} \ln(\tan(k^\circ))$$
- a. $-\frac{1}{2}\ln(3)$ b. $-\frac{1}{2}\ln(2)$ c. 0 d. $\frac{1}{2}\ln(2)$ e. $\frac{1}{2}\ln(3)$
16. Let $r_1 > r_2$ be the real solutions of the equation $x^2 + ax + b = 0$ and let $q_1 > q_2$ be the real solutions of the equation $x^2 - ax - b = 0$. Suppose $r_1 - r_2 = 7$ and $q_1 - q_2 = 1$. Determine the value of $|a| + |b|$.
- a. 3 b. 4 c. 9 d. 11 e. none of these
17. Let $\text{SPD}(n)$ be the sum of all positive divisors of n , where n is a positive integer. Let p be any prime number. What is the value of the ratio $\frac{\text{SPD}(2p)}{\text{SPD}(p)}$?
- a. 1 b. 2 c. $\frac{7}{3}$ d. 3 e. none of these
18. If $\frac{\sec^4 x}{a} - \frac{\tan^4 x}{b} = \frac{1}{a-b}$, then $\frac{\sec^8 x}{a^3} - \frac{\tan^8 x}{b^3} = ?$
- a. $\frac{1}{a^2 - b^2}$ b. $\frac{1}{(a-b)^2}$ c. $\frac{1}{a^3 - b^3}$ d. $\frac{1}{(a-b)^3}$ e. $\frac{1}{(a-b)^4}$

19. Determine the number of positive integers $x \leq 2024$ for which $x^2 - x$ is an integer multiple of 2024
- a. 1 b. 2 c. 8 d. 16 e. 32
20. In a magical swamp there are two species of talking amphibians: toads, whose statements are always true, and frogs, whose statements are always false. Four amphibians Alpha, Beta, Delta, and Epsilon live together in the swamp. They make the following statements:
- Alpha: "There is at least one frog."
Beta: "Delta is a frog."
Delta: "If you ask Beta, Beta would say that Epsilon is a frog."
Epsilon: "Alpha is a toad or Delta is a toad."
- How many of the amphibians are frogs?
- a. 0 b. 1 c. 2 d. 3 e. 4

SHORT ANSWER

Place the answer in the appropriate space.

66. Determine the sum of the reciprocals of the solutions of $x^4 + 8 = 24x^3 + 40x^2 + 16x$.

67. What is the least number of n consecutive positive integers, $n > 1$, that have a sum of 2024?

68. What is the sum of the last three digits of 9^{2024} ?

69. A square pyramid is formed from n balls such that 1 ball is at the top layer, 4 balls form a square on the second layer, 9 balls form a square on the third layer, and so on, so that each layer is a square array of balls supporting all previous layers. No such pyramid can be made of exactly 2024 balls. What is the next year for which that number of balls can be used in such a pyramid?

70. Define $f(n) = \begin{cases} \log_4(n), & \text{if } \log_4(n) \text{ is rational} \\ 0 & , \text{ otherwise} \end{cases}$, where n is any integer.

The summation below can be written in the form $\frac{p}{q}$, where p and q are integers such that $\gcd(p, q) = 1$.

$$\sum_{n=1}^{2024} f(n)$$

Calculate $p + q$.

2024 Wake Tech HS Comprehensive Test

1. D
2. B
3. C
4. B
5. D
6. D
7. D
8. A
9. B
10. C
11. E
12. E
13. A
14. D
15. A
16. D
17. E
18. D
19. C
20. B

66. 2

67. 11

68. 10

69. 2109

70. 57