

2014 State Math Contest Level II Test

1. The Bluebird zip line starts 45 feet above the ground and ends 6 feet above the ground. The horizontal distance covered by the zip line is 50 yards. Which of the following is the slope of the Bluebird zip line?
a. 0.3 ft per ft b. 0.9 ft per yd c. 0.26 ft per yd d. 0.78 ft per yd e. 0.78 ft

2. Let $S = \{3, 5, 7, 11, 13\}$. How many elements of the set S are factors of $2^{60} - 1$?
a. 1 b. 2 c. 3 d. 4 e. 5

3. In a sample of 5 positive data values, the median, the minimum, and the range are all equal. The mean equals one of the data values. What is the ratio of the maximum to the mean?
a. 1.2 b. 1.5 c. 1.6 d. 1.8 e. 2

4. Three vertices of a rectangle have coordinates $A(-16, 12)$, $B(-6, -2)$, $C(1, 3)$. Let D be the fourth vertex, then which of the rectangle's four vertices is farthest from the origin?
a. A b. B c. C d. D e. A and D

5. Sue's daughter, Sue's mother, and Sue were all born on Mother's Day. When Sue's daughter was born, Sue was $\frac{3}{7}$ the age of Sue's mother. Let A be Sue's age when her daughter was born. When Sue's daughter is $A + 4$ years old she will be exactly $\frac{1}{3}$ the age of Sue's mother. How old was Sue when her daughter was born?
a. 20 b. 24 c. 21 d. 27 e. 18

6. A shampoo company offers a 6.4 ounce bottle of shampoo and a 12.5 ounce bottle of shampoo in similar bottles. If the height of the smaller shampoo bottle is 8 inches, what is the height of the larger bottle?
- a. 18 in b. 16 in c. 14 in d. 12 in e. 10 in
7. The polynomial $3x^2 + 4xy - 4y^2$ can be factored as the product of two first degree polynomials. What is the sum of the two factors?
- a. $4x$ b. $4y$ c. $2x$ d. $2x + 2y$ e. $4x + 4y$
8. Twenty-three coins consisting of only quarters, dimes, and nickels are valued at \$2.55. If a different collection is made so that the new number of quarters is changed to the old number of nickels, the new number of dimes is changed to the old number of quarters, and the new number of nickels is changed to the old number of dimes, then the value would change to \$3.40. What was the original number of dimes?
- a. 4 b. 6 c. 8 d. 10 e. 12
9. Given a unit cube, what is the area of the largest equilateral triangle that will fit in the cube?
- a. $\sqrt{3}$ b. $2\sqrt{2}$ c. $\frac{\sqrt{2}}{2}$ d. $\frac{\sqrt{3}}{2}$ e. $\frac{\sqrt{6}}{2}$
10. If a , b , c , and d are nonzero numbers such that c and d are solutions of $x^2 + ax + b = 0$ and a and b are solutions of $x^2 + cx + d = 0$, then what is $a + b + c + d$?
- a. 2 b. 1 c. 0 d. -1 e. -2
11. If $x \neq 2y$ and $2x - 4y = 128y^3 - 16x^3$, then determine $x^2 + 2xy + 4y^2$.
- a. $\frac{1}{8}$ b. $-\frac{1}{8}$ c. $-\frac{1}{2}$ d. $\frac{1}{2}$ e. 2

12. Let k be the least positive integer that is divisible by each of the first six natural numbers. Determine the value of $\sqrt{\frac{k^2 + 3k}{15}}$.
- a. $6\sqrt{7}$ b. $2\sqrt{66}$ c. $\sqrt{66}$ d. 4 e. $12\sqrt{7}$
13. Points A and B have polar coordinates $\left(4, \frac{4\pi}{3}\right)$ and $(6, 0)$, respectively. What is the distance between the two points?
- a. $2\sqrt{19}$ b. $4\sqrt{5}$ c. $2\sqrt{13}$ d. $4\sqrt{3}$ e. $6\sqrt{2}$
14. A regular dodecahedron has 12 faces and 20 vertices. How many edges does it have?
- a. 36 b. 30 c. 24 d. 28 e. 26
15. The measure of an interior angle of a regular polygon is 8 times the exterior angle measure. How many sides does the polygon have?
- a. 12 b. 14 c. 16 d. 18 e. 20
16. Which of the following is a solution to the equation $\sqrt{4-x} + \sqrt{4+x} = x$?
- a. $2\sqrt{2}$ b. $\sqrt{3}$ c. $\sqrt{6}$ d. $2\sqrt{3}$ e. $3\sqrt{2}$
17. What is the last digit of 3^{2014} ?
- a. 1 b. 3 c. 5 d. 7 e. 9

18. Knaves always lie; knights always tell the truth. Al says, "Bo is a knight." Bo says, "Cy is a knave." Cy says, "Exactly one of Al and Bo is a knave." If Al, Bo, and Cy are each either a knight or a knave, which of the following must be true?
- a. Al is a knave and Cy is a knight.
 - b. Al is a knave and Cy is a knave.
 - c. Al is a knight and Cy is a knave.
 - d. Al is a knight and Cy is a knight.
 - e. It cannot be determined.
19. The solution to the equation $(\log_8 x^2)(\log_x 8)^2 = 1$ satisfies which inequality below?
- a. $0 < x \leq 1$
 - b. $1 < x \leq 10$
 - c. $10 < x \leq 50$
 - d. $50 < x \leq 100$
 - e. $x > 100$
20. A drawer contains two blue socks and four brown socks. If the power goes out and Bill must get dressed in the dark, what is the probability that the two socks he pulls out at random are a match?
- a. $\frac{1}{3}$
 - b. $\frac{7}{15}$
 - c. $\frac{2}{3}$
 - d. $\frac{1}{2}$
 - e. $\frac{2}{5}$
21. Eight girls get off a school bus that has a seating capacity of 64 children. When they leave, the ratio of girls to boys becomes 3:4. When 5 boys get on the bus, the ratio of girls to boys becomes 7:11. How many students were initially on the bus?
- a. 55
 - b. 53
 - c. 52
 - d. 62
 - e. 57
22. Let S be the set of all points in the plane which are twice as far from the point $(3, -1)$ as from the point $(4, 2)$. The points of S lie on what type of curve?
- a. Line
 - b. Circle
 - c. Parabola
 - d. Hyperbola
 - e. Ellipse

23. How many ordered pairs of positive integers (a,b) satisfy the equation $\sqrt{a+\sqrt{b}} = \sqrt{7} + \sqrt{7}$?
- a. 27 b. 29 c. 31 d. 25 e. 28
24. How many times in a 24-hour day are the minute hand and the hour hand perpendicular to each other?
- a. 40 b. 42 c. 44 d. 46 e. 48
25. Three friends joined a weight loss program. The oldest lost 10 pounds more than Kathy. The combined weight loss of Shirley and the youngest was 20 pounds more than Tim's loss. If Shirley is not the oldest, how many pounds did she lose?
- a. 20 b. 30 c. 0 d. 10 e. 25

SHORT ANSWER

Place the answer in the appropriate space.

66. Determine $\log\left(\log\left(\log\left(10^{10,000,000,000}\right)\right)\right)$.

67. Let a and b be integers such that $a \leq b$. Their arithmetic mean is 6 more than their geometric mean. If $a = 48$, what is the value of b ?

68. A bus company can be chartered by a group for trips from Raleigh to Washington, D.C. The cost per person for a group of 16 is \$42. For each additional person over 16, the cost per person fee is reduced by \$1.50. What number of people would maximize the revenue for the bus company?

69. The mean of three numbers is ten more than the least of the three and fifteen less than the greatest of the three. If the median of the three numbers is 5, what is their sum?

70. What is the sum of the solutions to the equation $3^{2x+1} - 3^{x+3} - 3^x + 3^2 = 0$?

Answer Key

1. d
2. e
3. c
4. a
5. b
6. e
7. a
8. c
9. d
10. e
11. b
12. a
13. a
14. b
15. d
16. d
17. e
18. c
19. d
20. b
21. e
22. e
23. a
24. c
25. b

66. 1
67. 108
68. 22
69. 30
70. 1