Level II
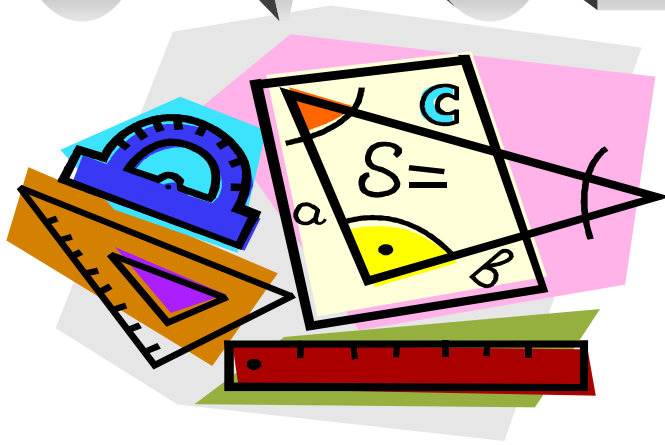
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March 22, 2018

1. During a recent police investigation, Chief Inspector Stone was interviewing five local villains to try and identify who stole Mrs. Archer's cake from the fair. Below is a summary of their statements:

   Arnold: "It wasn't Edward." "It was Brian."

   Brian: "It wasn't Charles." "It wasn't Edward."

   Charles: "It was Edward." "It wasn't Arnold."

   Derek: "It was Charles." "It was Brian."

   Edward: "It was Derek." "It wasn't Arnold"

   It was well known that each suspect told exactly one lie. Which suspect did it?

   a. Edward   b. Derek   c. Charles   d. Brian   e. Arnold

2. How many lines are determined by six distinct points on a circle?

   a. 15   b. 30   c. 9   d. 18   e. 36

3. The line $y = 2x - 6$ is translated four units right, then six units down, and then reflected about the line $x = y$. What is the equation of the transformed line?

   a. $y = \dfrac{1}{2}x + 7$ b. $y = -\dfrac{1}{2}x - 4$ c. $y = -2x + 20$ d. $y = -\dfrac{1}{2}x - 10$ e. $y = \dfrac{1}{2}x + 10$

4. Two sides of an obtuse triangle have lengths 4 cm and 8 cm. How many triangles are possible if the length of the third side is also an integer?

   a. 4   b. 5   c. 6   d. 7   e. 8

5. To increase the area of a circle by 44%, by what percentage must you increase the radius?

    a. 20%   b. 22%   c. 25%   d. 22.5%   e. 18%

6. Convex quadrilateral $MATH$ has the following measurements: $MA = 20$, $AT = 15$, $TH = 8$, $MT = 25$, and $AH = 17$ What is the area of $MATH$?

    a. 180   b. 210   c. 205   d. 220   e. 185

7. Philip rides his bike on a road with rolling hills and no flat stretches. He rides uphill at a constant rate of 5 mph and rides downhill at a constant rate of 15 mph. He starts and ends his 40 mile, round trip ride at the same place. What was his average speed for the entire trip?

    a. 10 mph   b. 8 mph   c. 6 mph   d. 7.5 mph   e. 8.5 mph

8. The angles of a triangle are all prime numbers. What is the measure of the largest possible angle?

    a. 169°   b. 171°   c. 173°   d. 177°   e. 179°

9. The mean of five positive integers is 7, their median is 8, and their mode is 9. What is the largest possible range of such a set?

    a. 3   b. 4   c. 5   d. 6   e. 7

10. Compute the area of the triangle whose vertices are the y-intercept, the positive x-intercept, and the vertex of the graph of $y = -\dfrac{3}{4}|x-4|+12$.

    a. 40   b. 36   c. 42   d. 48   e. 45

11. On a car, a particular brand of tires gets 40,000 miles on a front wheel or 60,000 miles on a rear wheel. By interchanging the front and rear tires once what is the greatest distance in miles that one could get from the set of four tires?

   a. 52,000   b. 50,000   c. 48,000   d. 40,000   e. 44,000

12. A builder needs 10,000 bricks to finish a job. He is sure from long experience that no more than 7% of a load of bricks is broken on delivery. If bricks are sold only in lots of 100, what is the minimum number of bricks he should order to be sure of having enough to finish the job?

   a. 10,900   b. 10,600   c. 11,000   d. 10,700   e. 10,800

13. A floor tile has the shape of a regular polygon. If the tile is removed from the floor and rotated through 50° it will fit back exactly into its original place in the floor. What is the least number of sides that the polygon can have?

   a. 36   b. 24   c. 25   d. 30   e. 8

14. $WXYZ$ is a square and P is a point in its interior such that $PW = PZ = PV = 10$ in, where V is a point on side $\overline{XY}$ and $\overline{PV} \perp \overline{XY}$. What is the area of $WXYZ$ in square inches?

   a. 225   b. 232   c. 248   d. 256   e. 324

15. A cube measuring 6 inches on an edge is painted red. The cube is then cut into cubes that measure 3 inches on an edge and the unpainted faces are painted green. Finally, these cubes are cut into unit cubes and their unpainted faces are painted blue. How many faces are painted blue on the unit cubes?

   a. 972   b. 756   c. 648   d. 432   e. 864

16. Given $f(x)=|x|-x$, then what are the coordinates of the midpoint of the line segment joining the points $\big(f(2),f(-2)\big)$ and $\big(f(4),f(-4)\big)$?

   a. $(0,0)$ b. $(0,6)$ c. $(3,6)$ d. $(0,4)$ e. $(0,3)$

17. The line $y=mx+b$ is tangent to the circle $(x+1)^2+(y-1)^2=25$ at $(3,4)$. What is $m+b$?

   a. $\dfrac{5}{12}$ b. $\dfrac{5}{2}$ c. $\dfrac{7}{2}$ d. $\dfrac{20}{3}$ e. $\dfrac{35}{4}$

18. The area of the four-sided region in the first quadrant bounded by the x-axis, y-axis, and the lines $3x+4y=12$ and $2y-x=2$ is cut in half by the line $y=kx$. What is k?

   a. $\dfrac{33}{76}$ b. $\dfrac{2}{5}$ c. $\dfrac{11}{19}$ d. $\dfrac{1}{2}$ e. $\dfrac{21}{38}$

19. Assume the measures of the three angles of a triangle in degrees are integers. How many of these triangles exist such that the measures of all three angles are perfect squares?

   a. 0   b. 1   c. 2   d. 3   e. 4

20. A group of students visited a museum. They entered through doorway P and departed through doorway Q. In between, they passed through each doorway once and only once, except for one doorway. Which doorway did they not pass through?
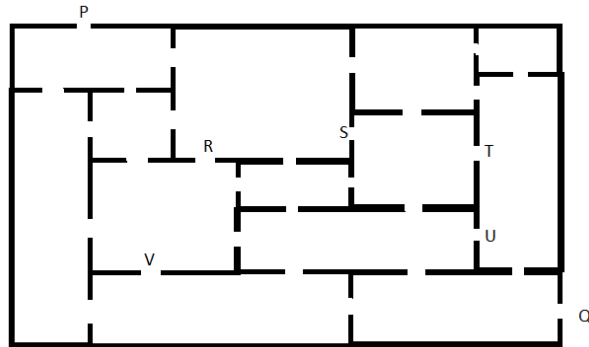
a. R   b. S   c. T   d. U   e. V

SHORT ANSWER
Place the answer in the appropriate space.

66. The area of a triangle is 12 square inches and one of its sides is 8 inches. What is the minimum perimeter of the triangle?

67. An isosceles triangle has an area of $196\sqrt{63}$. If the two congruent sides are each four times the base, what is the perimeter of the triangle?

68. Fence posts are placed 4 feet apart. How many posts will be needed to enclose a rectangular yard that is 24 yards by 36 yards?

69. What is the area enclosed by the graph of $|x-2|+|y+3|=4$?

70. A stocker at a bakery counted the number of cupcakes that were still on the shelf at the end of the day. Because the stocker was in a hurry to leave, the number written down was missing its units digit. The next morning the owner was surprised to find that she had 89 more cupcakes on the shelf than the stocker had written. What was the missing digit?

1. C
2. A
3. E
4. B
5. A
6. B
7. D
8. C
9. E
10. D
11. C
12. E
13. A
14. D
15. E
16. B
17. D
18. A
19. B
20. C

66. 18
67. 252
68. 90
69. 32
70. 8