Comprehensive
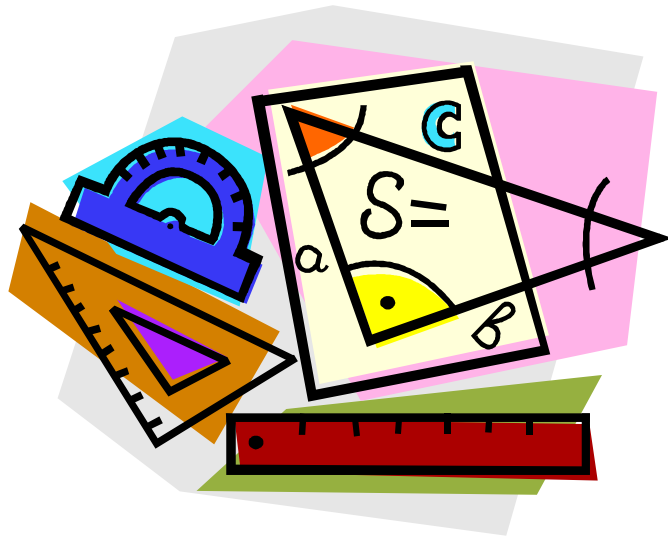

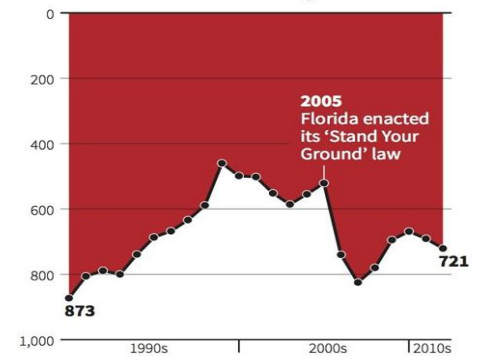# Do NOT open until you are told to do so.

March 23, 2017

1. In 2005 the state of Florida enacted the "Stand Your Ground Law". Which of the following statements are true based on the graph from the Florida Department of Law Enforcement?

Gun deaths in Florida
Number of murders committed using firearms

   i. There were fewer murders committed using firearms in 2006 than in the previous year.

   ii. The lowest number of murders committed using firearms occurred in the late 1990s.

   iii. There were 873 murders committed using firearms in 1990.

   a. i and iii   b. i   c. ii and iii   d. iii   e. i, ii, and iii

2. In a series of 5 games to be played between two equally matched teams, the first team to win three games becomes the champion. Team A has won the first game. What is the probability that Team A will be the champion?

   a. $\dfrac{11}{16}$ b. $\dfrac{3}{5}$ c. $\dfrac{1}{2}$ d. $\dfrac{13}{16}$ e. $\dfrac{9}{16}$

3. A bicyclist rides 8 miles on a straight road from point A to point B. She then makes a sharp 60 degree right turn (as indicated in the diagram) onto another straight road and rides 5 miles to point C. At point C she gets a flat tire. Dreading the 13 mile hike home, she is delighted when she realizes that there is a straight path back home to point A. How far would she have to hike on the path to get home?
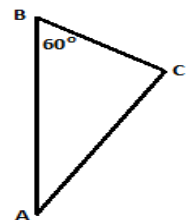
   a. $6\sqrt{3}$ mi   b. 7 mi   c. 8.5 mi   d. $5\sqrt{3}$ mi   e. $4\sqrt{3}$ mi

4. What is the graph of $x^2 + xy + x + 3y = 6$?

   a. an ellipse   b. a parabola   c. a hyperbola   d. 2 parallel lines   e. 2 intersecting lines

5. Copiers A, B, and C are used to produce $3n$ copies, n on each copier. Copier A makes 18 copies per minute and copier B makes 9 copies per minute. If the average copy speed is 15 copies per minute, what is the rate in copies per minute at which copier C makes copies?

    a. 21   b. 24   c. 25   d. 30   e. 36

6. What is the sum of the squares of the sines of the three angles of right triangle $\triangle ABC$?

    a. 2   b. $\sqrt{2}$ c. 1   d. $1+\sqrt{2}$ e. not determinable

7. How many positive integers from 1 to 400, inclusive, have exactly three positive integral factors?

    a. 6   b. 5   c. 4   d. 7   e. 8

8. $\arccos\left(\cos\left(-\dfrac{\pi}{3}\right)\right) - \arccos\left(\cos\left(\dfrac{4\pi}{3}\right)\right)$

    a. $\dfrac{\pi}{3}$ b. $-\dfrac{2\pi}{3}$ c. $\dfrac{2\pi}{3}$ d. π e. $-\dfrac{\pi}{3}$

9. What is the value of $C+D$ that would produce more than one solution to the system of equations
    $$\begin{cases} x+2y-z=3 \\ 3x-y+2z=-2 \\ -3x+8y+Cz=D \end{cases} ?$$

    a. 3   b. 4   c. 5   d. 6   e. 8

10. In a right triangle $\triangle ABC$, $m\angle A = 90°$ and $m\angle C = 30°$. Point D lies on $\overline{AC}$ such that $\overline{BD}$ bisects $\angle ABC$. What is $\dfrac{DC}{BC}$?

   a. $\sqrt{3}$ b. $\dfrac{1}{2}$ c. $\dfrac{\sqrt{3}}{2}$ d. $2\sqrt{3}$ e. $\dfrac{\sqrt{3}}{3}$

11. Six cylinders of radius 1 foot are stacked in a pile with three on the bottom, then two, and topped by one as seen in the diagram. What is the height of the pile?
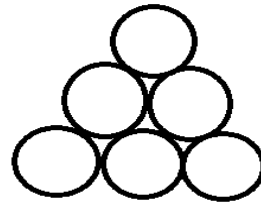
   a. $1+\sqrt{3}$ ft   b. $\dfrac{3+\sqrt{2}}{2}$ ft   c. $2+2\sqrt{3}$ ft   d. $\dfrac{2+\sqrt{3}}{2}$ ft   e. $3+2\sqrt{3}$ ft

12. The speed of light is approximately 186,000 miles per second. A nanosecond is one billionth of a second. Which of the following most closely approximates the distance traveled by light in a nanosecond?

   a. 6 in   b. 1 ft   c. 2 ft   d. 1.5 ft   e. 8 in

13. In a public park, a triangular region of grass is bounded by walkways that are 8 ft, 7 ft, and 3 ft long. Sprinkler heads are placed at the vertices of the triangular region and set so that the angles they rotate through will keep the walkways dry. What is the size of the angle of rotation (in degrees) between the 8 ft and 3 ft sides?

   a. $60°$ b. $\arccos(0.25)$ c. $30°$ d. $\arccos(0.65)$ e. $\arccos(0.45)$

14. Given $f(x)+f(x+7)=6x-5$ and $f(5)+f(19)=34$, what is $f(12)$?

    a. 29   b. 31   c. 23   d. 27   e. 28

15. A square is inscribed in a circle which is inscribed in a square which is inscribed in a circle which is inscribed in a square! What is the ratio of the area of smallest square to the area of the largest square?

    a. $\dfrac{\pi}{9}$ b. $\dfrac{\pi}{6}$ c. $\dfrac{1}{8}$ d. $\dfrac{1}{4}$ e. $\dfrac{1}{2\sqrt{2}}$

16. A toddler has 2 blue blocks, 2 red blocks, 1 purple block, and 1 yellow block. If the blocks are stacked vertically, how many different stacks of 3 blocks can the toddler make?

    a. 40   b. 50   c. 56   d. 48   e. 42

17. The matrix $A = \begin{bmatrix} a & 8 \\ -3 & b \end{bmatrix}$ is its own multiplicative inverse. What is $|a-b|$?

    a. 0   b. 6   c. 8   d. 10   e. 12

18. Ed needs to transport 49 guests to a wedding. Cars hold 5 guests and rent for $29 each, while vans hold 7 guests and rent for $41 each. What is the minimum cost to transport the 49 guests?

    a. $280   b. $285   c. $282   d. $287   e. $290

19. Tom and Tim play the following game: Tom draws a chip from a bag containing 3 green chips and 6 red chips. If the chip is green, Tom wins; if not, Tom keeps the chip, and Tim draws a chip. If the chip is red, Tim wins; otherwise, Tim keeps the chip, and the alternating draws repeat. What is the probability that Tom wins the game?

a. $\dfrac{5}{12}$ b. $\dfrac{1}{3}$ c. $\dfrac{1}{4}$ d. $\dfrac{1}{2}$ e. $\dfrac{7}{12}$

20. What is the area of the triangle whose side lengths are the solutions of $x^3 - 18x^2 + 105x - 200 = 0$?

a. 18   b. 16   c. 12   d. 20   e. 10

21. Let $f(x) = \ln(6-x)$ and $g(x) = |x^2 - 10x + 15|$. The domain of $f(g(x))$ can be written in interval notation as $(a,b) \cup (c,d)$. What is $b + d$?

a. 6   b. 11   c. 8   d. 12   e. 10

22. Suppose that $\sec \alpha = \dfrac{5}{3}$ with $\dfrac{3\pi}{2} < \alpha < 2\pi$ and $\sin \beta = -\dfrac{12}{13}$ with $\pi < \beta < \dfrac{3\pi}{2}$. Calculate the exact value of $\cos(\alpha - \beta)$.

a. $\dfrac{53}{65}$ b. $\dfrac{33}{65}$ c. $-\dfrac{63}{65}$ d. $\dfrac{63}{65}$ e. $-\dfrac{33}{65}$

23. A heifer weighing 200 lb today gains 5 lbs per day with a food cost of 45 cents per day. The price for heifers is 65 cents per pound, but is falling one cent per day. How many days after today should the heifer be sold to maximize profit?

a. 10   b. 6   c. 8   d. 12   e. 9

24. Recall the following Roman Numeral values: VII = 7: IX = 9; XXV = 25; XLVI = 46; XCII = 92; and MDCLXI = 1661. Compute MMXLVII ÷ LXXXIX.

    a. XXIV   b. XXIII   c. XLII   d. XXXIV   e. XXVI

25. Five sentences were written on the board in a logic class as a quiz, but someone erased the last two statements to keep the instructor from giving the quiz. Instead the instructor said you still have to take the quiz. It consists of one question – "How many of the original five statements were true?"

    Statement 1: Statement 2 is true.

    Statement 2: At most, one of these five statements is true.

    Statement 3: All five statements are true.

    Statement 4:

    Statement 5:

    a. 4   b. 3   c. 2   d. 1   e. 0

SHORT ANSWER
Place the answer in the appropriate space.

66. The triangles ΔABC and ΔDEF are not isosceles, not congruent, and have integer length sides. If they have the same perimeter, what is the smallest such perimeter they could share?

67. The centers of circles defined by $x^2 + y^2 + 4x - 4y + 4 = 0$, $x^2 + y^2 - 10x - 10y + 25 = 0$, and $x^2 + y^2 - 6x + 6y + 9 = 0$ are joined to form a triangle. What is the area of the triangle?

68. Assuming ties are allowed, what is the number of ways four contestants can be ranked, if the order of tied contestants doesn't matter?

69. Tristan took jugs of milk to the market. He first sold three-fourths of his jugs plus a fourth of a jug to Ann. Then he sold three-fourths of his remaining jugs plus a fourth of a jug to Beth. Finally, he sold three-fourths of his remaining jugs plus a fourth of a jug to Cindy. He was left with 3 jugs. How many jugs did Tristan begin with?

70. John rolls a fair six-sided die and observes the result. Paul is challenged to roll the die until matching John's result. What is the probability that it will take Paul at least four tosses until success? Write your answer as a reduced fraction.

Answer Key

1. C
2. A
3. B
4. E
5. D
6. A
7. E
8. E
9. D
10. E
11. C
12. B
13. A
14. A
15. D
16. E
17. D
18. B
19. A
20. C
21. D
22. B
23. C
24. B
25. C

66. 13
67. 25
68. 75
69. 213
70. $\dfrac{125}{216}$