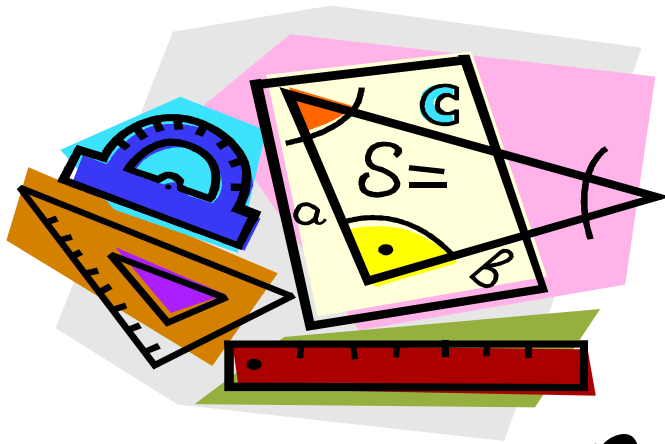


Level III



**Do NOT open until
you are told to do so.**

March 22, 2018

1. During a recent police investigation, Chief Inspector Stone was interviewing five local villains to try and identify who stole Mrs. Archer's cake from the fair. Below is a summary of their statements:

Arnold: "It wasn't Edward." "It was Brian."

Brian: "It wasn't Charles." "It wasn't Edward."

Charles: "It was Edward." "It wasn't Arnold."

Derek: "It was Charles." "It was Brian."

Edward: "It was Derek." "It wasn't Arnold"

It was well known that each suspect told exactly one lie. Which suspect did it?

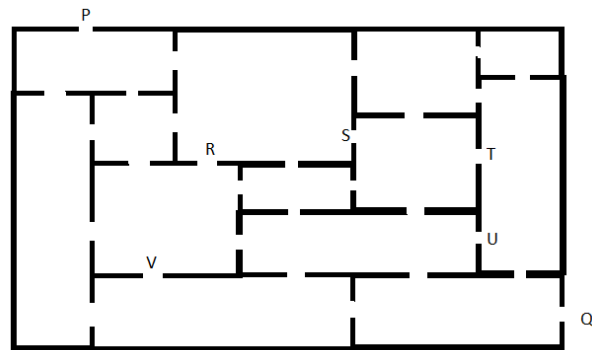
- a. Derek b. Charles c. Edward d. Brian e. Arnold
2. What is $x + y$, if $x^{\frac{2}{3}} = 8^{\frac{2}{y}}$ and x and y are positive integers greater than one?
- a. 9 b. 3 c. 2 d. 11 e. infinite number of solutions
3. To increase the area of a circle by 44%, by what percentage must you increase the radius?
- a. 20% b. 22% c. 25% d. 22.5% e. 18%
4. Let $n = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ where a , b , and c are all positive integers. What is the largest value of n that is less than 1?
- a. $\frac{9}{10}$ b. $\frac{11}{12}$ c. $\frac{19}{20}$ d. $\frac{41}{42}$ e. $\frac{63}{64}$

5. The angles of a triangle are all prime numbers. What is the measure of the largest possible angle?
- a. 169° b. 171° c. 173° d. 177° e. 179°
6. A box contains 30 pieces of candy that are yellow, blue, or red. If 25 pieces of candy are randomly selected, there will always be at least 3 red, at least 5 blue, and at least 7 yellow pieces. How many yellow pieces of candy were originally in the box?
- a. 12 b. 8 c. 10 d. 7 e. 15
7. Suppose $f(x) = \frac{x^2 - 16}{ax + b}$ for some real numbers a and b , and that $f(x)$ has an oblique asymptote of $y = 3x + 7$. What is $f(-3)$?
- a. $\frac{65}{9}$ b. $\frac{63}{16}$ c. $\frac{35}{9}$ d. $-\frac{7}{16}$ e. $-\frac{35}{9}$
8. The line $y = mx + b$ is tangent to the circle $(x+1)^2 + (y-1)^2 = 25$ at $(3, 4)$. What is $m + b$?
- a. $\frac{5}{12}$ b. $\frac{5}{2}$ c. $\frac{7}{2}$ d. $\frac{20}{3}$ e. $\frac{35}{4}$
9. Suppose $\log_A B = \log_B A$, where $A > B > 1$. Which of the following is true?
- a. $AB = 2$ b. $AB = e$ c. $AB = 4$ d. $AB = e^2$ e. This is not possible.
10. The area of the four-sided region in the first quadrant bounded by the x -axis, y -axis, and the lines $3x + 4y = 12$ and $2y - x = 2$ is cut in half by the line $y = kx$. What is k ?
- a. $\frac{33}{76}$ b. $\frac{2}{5}$ c. $\frac{11}{19}$ d. $\frac{1}{2}$ e. $\frac{21}{38}$

11. A cube measuring 6 inches on an edge is painted red. The cube is then cut into cubes that measure 3 inches on an edge and the unpainted faces are painted green. Finally, these cubes are cut into unit cubes and their unpainted faces are painted blue. How many faces are painted blue on the unit cubes?
- a. 972 b. 756 c. 648 d. 432 e. 864
12. Compute the following: $\sum_{n=1}^{2018} (i^n + i^{-n})$
- a. 2 b. -2 c. 0 d. $i-2$ e. $2-i$
13. A skydiver free-falls at 120 mph and falls with his parachute open at 15 mph. If the diver opened his chute half way down, and the total time of descent was 6 minutes, how high in miles was the plane when he jumped?
- a. 1 b. $\frac{20}{3}$ c. $\frac{8}{3}$ d. $\frac{11}{3}$ e. 2
14. How many real solutions does $2^x - 2^{-x} = 2^2$ have?
- a. 0 b. 1 c. 2 d. 3 e. 4
15. A total of 120 students were surveyed about their shopping habits: 65% shop for new clothes, 50% shop for school supplies, and 40% shop for clothes and school supplies. How many students shop for clothes or school supplies?
- a. 100 b. 95 c. 90 d. 85 e. 80

16. Let $A = \begin{bmatrix} x & 4 \\ 6 & y \end{bmatrix}$. If x and y are integers and $\det(A^2) = 1$, which of the following is a possible value of $x + y$?
- a. 14 b. 17 c. 20 d. 23 e. 26
17. Compute the area of the smallest region bounded by the graph of $y = |x|$ and $x^2 + y^2 = 25$.
- a. $\frac{25\pi}{4}$ b. $\frac{25\pi}{2}$ c. 25π d. $\frac{75\pi}{4}$ e. 5π
18. The numbers p, q, r, s , and t are consecutive positive integers, arranged in increasing order. If $p + q + r + s + t$ is a perfect cube and $q + r + s$ is a perfect square, then what is the smallest possible value of r ?
- a. 75 b. 288 c. 225 d. 675 e. 725
19. One of the Roman dice in the British Museum has 6 square faces and 8 triangular faces. It is twice as likely to land on any given square face as any given triangular face. What are the odds of landing on a triangular face if this particular die is thrown?
- a. $\frac{2}{5}$ b. $\frac{3}{4}$ c. $\frac{4}{3}$ d. $\frac{3}{2}$ e. $\frac{2}{3}$

20. A group of students visited a museum. They entered through doorway P and departed through doorway Q. In between, they passed through each doorway once and only once, except for one doorway. Which doorway did they not pass through?



- a. R b. S c. T d. U e. V

SHORT ANSWER

Place the answer in the appropriate space.

66. Let M be the greatest integer less than 30 such that $\frac{M!(M+1)!}{2}$ is a perfect square. Let N be the greatest integer that divides $c^4 - c^2$ for all integers $c > 1$. What is $M + N$?
67. How many positive integers less than or equal to 1000 have an equal number of even and odd factors? For example, 10 would be counted since it has two odd (1 and 5) and two even (2 and 10) factors.
68. Consider a data set that consists of positive integers less than 51. There is exactly one 1 in the data set, and every other integer appears twice as many times as its predecessor appears (so there are two 2s and four 3s). What is the median of this data set?
69. What is the sum of all the distinct positive five digit numbers that can be formed by permuting the digits 1, 2, 3, 4, and 5?
70. Some hikers set out on a hike at noon. At some point, they turn around and follow the same path back to where they began, and arrive there at 8:00 p.m. Their speed is 4 mph on level ground, 3 mph uphill, and 6 mph downhill. How many miles did they hike before turning around?

Answer Key

1. B
2. D
3. A
4. D
5. C
6. A
7. B
8. D
9. E
10. A
11. E
12. B
13. C
14. B
15. C
16. E
17. A
18. D
19. E
20. C

66. 29
67. 250
68. 50
69. 3,999,960
70. 16