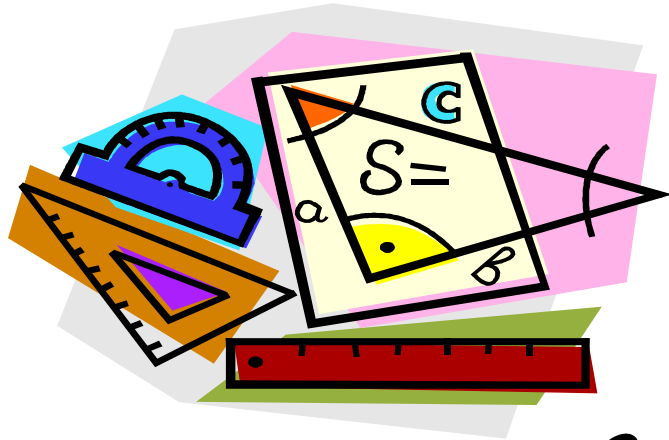


Level 11



**Do NOT open until
you are told to do so.**

March 22, 2018

1. During a recent police investigation, Chief Inspector Stone was interviewing five local villains to try and identify who stole Mrs. Archer's cake from the fair. Below is a summary of their statements:

Arnold: "It wasn't Edward." "It was Brian."

Brian: "It wasn't Charles." "It wasn't Edward."

Charles: "It was Edward." "It wasn't Arnold."

Derek: "It was Charles." "It was Brian."

Edward: "It was Derek." "It wasn't Arnold"

It was well known that each suspect told exactly one lie. Which suspect did it?

- a. Arnold b. Brian c. Charles d. Derek e. Edward

2. The solution to the system of equations $\begin{cases} y = 2x + 1 \\ 3x - y = 4 \end{cases}$ is the ordered pair (a, b) . What is $a + b$?

- a. 13 b. 16 c. 10 d. 4 e. 8

3. When each is given in Fahrenheit, the sum of the high and low temperature for Memphis, TN, on a particular day is 68° . What is the sum of the high and low temperatures in Memphis on the same day if each is given in Celsius?

- a. $\frac{20}{9}$ b. $\frac{36}{5}$ c. 20 d. $\frac{772}{9}$ e. $\frac{772}{5}$

4. For real numbers a and b , define an operation Δ as $a\Delta b = ab^2 - |a|$. Calculate $[(-2)\Delta 5]\Delta(-1)$.

- a. 0 b. -96 c. -53 d. -104 e. 96

5. By what percentage must a storeowner first raise the price of an item that he plans to discount by 20 percent in order to keep his profit the same?
- a. 12.5% b. 20% c. 30% d. 22.5% e. 25%
6. In base b , where b is a positive integer, $2000_b - 320_b = 1130_b$. What is the value of b ?
- a. 5 b. 4 c. 6 d. 8 e. 12
7. The mean of five positive integers is 7, their median is 8, and their mode is 9. What is the smallest possible range of such a set?
- a. 3 b. 4 c. 5 d. 6 e. 7
8. The line $y = 2x - 6$ is translated four units right, then six units down, and then reflected about the line $x = y$. What is the equation of the transformed line?
- a. $y = \frac{1}{2}x + 7$ b. $y = -\frac{1}{2}x - 4$ c. $y = -2x + 20$ d. $y = -\frac{1}{2}x - 10$ e. $y = \frac{1}{2}x + 10$
9. Which of the following is equivalent to $\frac{3^{2020} - 3^{2019}}{18}$?
- a. 3^{2016} b. 3^{2017} c. 3^{2015} d. $2(3^{2018})$ e. $2(3^{2015})$

10. The angles of a triangle are all prime numbers. What is the measure of the largest possible angle?
- a. 169° b. 171° c. 173° d. 177° e. 179°
11. A normal duck has two legs. A lame duck has one leg. A sitting duck has no legs. There are 33 ducks with a total of 32 legs. The total number of normal ducks and lame ducks is twice the number of sitting ducks. How many lame ducks are there?
- a. 9 b. 10 c. 11 d. 12 e. 13
12. What is $x + y$, if $x^{\frac{2}{3}} = 8^{\frac{2}{y}}$ and x and y are positive integers greater than one?
- a. 11 b. 3 c. 9 d. 2 e. infinite number of solutions
13. Let $n = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ where a , b , and c are all positive integers. What is the largest value of n that is less than 1?
- a. $\frac{9}{10}$ b. $\frac{11}{12}$ c. $\frac{19}{20}$ d. $\frac{41}{42}$ e. $\frac{63}{64}$
14. A 7 foot long log is being cut into smaller logs. The time t in minutes it takes to saw through the log increases as the radius r in inches of the log increases: $t = \frac{r}{2} + 1$. The radius of the log increases as the distance d in feet from the smaller end increases: $r = d + 10$. What is the fewest number of minutes it takes to cut 2 logs that are 2 feet long and 3 logs that are 1 foot long?
- a. 27.5 b. 29.5 c. 31 d. 32.5 e. 39

15. Philip rides his bike on a road with rolling hills and no flat stretches. He rides uphill at a constant rate of 5 mph and rides downhill at a constant rate of 15 mph. He starts and ends his 40 mile, round trip ride at the same place. What was his average speed for the entire trip?
- a. 10 mph b. 7.5 mph c. 6 mph d. 8.5 mph e. 8 mph
16. Compute the area of the triangle whose vertices are the y -intercept, the positive x -intercept, and the vertex of the graph of $y = -\frac{3}{4}|x - 4| + 12$.
- a. 36 b. 40 c. 42 d. 45 e. 48
17. On a car, a particular brand of tires gets 40,000 miles on a front wheel or 60,000 miles on a rear wheel. By interchanging the front and rear tires once what is the greatest distance in miles that one could get from the set of four tires?
- a. 48,000 b. 50,000 c. 52,000 d. 40,000 e. 44,000
18. A builder needs 10,000 bricks to finish a job. He is sure from long experience that no more than 7% of a load of bricks is broken on delivery. If bricks are sold only in lots of 100, what is the minimum number of bricks he should order to be sure of having enough to finish the job?
- a. 10,900 b. 10,600 c. 11,000 d. 10,700 e. 10,800
19. A box contains 30 pieces of candy that are yellow, blue, or red. If 25 pieces of candy are randomly selected, there will always be at least 3 red, at least 5 blue, and at least 7 yellow pieces. How many yellow pieces of candy were originally in the box?
- a. 7 b. 8 c. 10 d. 12 e. 15

- 5

SHORT ANSWER

Place the answer in the appropriate space.

66. What is the sum of all real solutions to the equation $\left|4 - \left|3 - \left|2 - \left|1 - x\right|\right|\right|\right| = 0$?
67. Winnie has won 30 out of 50 games of a computer solitaire game. What is the fewest number of additional games that she must play (and win) to increase her success rate to 65%? Assume Winnie rounds her success rate to the nearest 1%.
68. What is the sum of the smallest positive integers a and c such that $12a + 5b = 10$ and $4b + c = 8$?
69. There are 150 socks in a bin: 30 blue, 10 pink, 20 green, 40 black, and 50 white. Jerry randomly pulls socks out of the drawer (without looking), one at a time, and does not replace them. Let m be the minimum number of socks that he would need to pull out to guarantee that he has at least one matching (same color) pair. Let M be the minimum number of socks that he must pull out to guarantee that he has at least one sock of each color. What is the product of m and M ?
70. For all values of x , $(ax + 3)(bx + 5) = 12x^2 + cx + 15$. If $a + b = 8$, what is the sum of all possible values of c ?

Answer Key

1. C
2. B
3. A
4. D
5. E
6. A
7. C
8. E
9. B
10. C
11. D
12. A
13. D
14. B
15. B
16. E
17. A
18. E
19. D
20. C

66. 2
67. 7
68. 53
69. 846
70. 64