

2016 State Math Contest Level III Test

1. A certain disease occurs in 8% of the male population and the test for it is 80% accurate (which means 80% of the time the test correctly identifies who does or who does not have the disease). If a man tests positive which of the following is closest to the chance that he truly has the disease?
a. 80% b. 50% c. 30% d. 25% e. 20%

2. A collection of 41 coins is worth exactly \$1.00. If it has at least one quarter, one dime, one nickel, and one penny, what is the total number of dimes and nickels?
a. 5 b. 6 c. 7 d. 8 e. 9

3. Let $f(x) = x^2$. Define $P(a, b)$ to be the y -coordinate of the y -intercept of the line through $(a, f(a))$ and $(-b, f(-b))$. Determine $P(4, 6) + P(3, 5)$.
a. 30 b. 33 c. 36 d. 39 e. 42

4. A three digit number, ABC, is 29 times the sum of its digits. When the first digit is made the last digit the new number, BCA, is 68 times the sum of the digits. When the new first digit is made the last digit, the new number, CAB, is 14 times the sum of the digits. What is the sum of the squares of the digits?
a. 38 b. 41 c. 43 d. 37 e. 39

5. The polynomial $a^4 + b^4$ is equal to $X(a+b)^4 - Y(a+b)^2(ab) + Z(ab)^2$. What is $X + Y + Z$?
a. 4 b. 5 c. 6 d. 7 e. 8

6. What is the sum of the set of numbers $\left\{-64^{\frac{1}{3}}, 64^{-\frac{2}{3}}, 64^{\frac{2}{3}}, 64^{-\frac{1}{3}}, -64^{\frac{1}{2}}\right\}$?
a. 4 b. $\frac{61}{16}$ c. $-\frac{63}{16}$ d. 0 e. $\frac{69}{16}$

7. Mabel throws two fair, six-sided dice, a red one and a white one. What is the probability that the red one beats the white one?
- a. $\frac{1}{2}$ b. $\frac{1}{6}$ c. $\frac{5}{12}$ d. $\frac{7}{12}$ e. $\frac{1}{3}$
8. The numbers one through twelve are written in two rows and six columns so that the 2 row sums are equal to one another, and the 6 column-sums are also equal to one another. If the number eight appears in the first row, then how many even numbers are there in the second row?
- a. 1 b. 2 c. 3 d. 4 e. 5
9. For how many rational values of x is $P(x) = |x^2 - 28x + 160|$ a prime number?
- a. 0 b. 1 c. 2 d. 3 e. 4
10. Three faces of a rectangular box have a common point, which is a corner of the box. The centers of these faces are the vertices of a triangle with sides of length 4, 5, and 6 cm. What is the volume of the box?
- a. $120\sqrt{2} \text{ cm}^3$ b. $90\sqrt{6} \text{ cm}^3$ c. $45\sqrt{3} \text{ cm}^3$ d. 125 cm^3 e. $45\sqrt{6} \text{ cm}^3$
11. How many of the following seven shapes have rotational symmetry with a positive angle measure less than 360° : rectangle, parallelogram, trapezoid, regular pentagon, triangle, circle, and hexagon?
- a. 2 b. 3 c. 4 d. 5 e. 6

12. If you roll three fair six-sided dice, what is the probability that the sum of the three numbers rolled is prime?

- a. $\frac{73}{216}$ b. $\frac{2}{3}$ c. $\frac{1}{3}$ d. $\frac{37}{108}$ e. $\frac{71}{216}$

13. What is the sum of the solutions of $3\sqrt{2-x} = x-4$?

- a. 1 b. 2 c. -1 d. -2 e. no solutions

14. If a and b are real numbers such that $0 < a < b$ and $a^2 + b^2 = 6ab$, then what is the value of $\frac{a+b}{a-b}$?

- a. -2 b. -1 c. $\sqrt{2}$ d. $-\sqrt{2}$ e. $-\sqrt{6}$

15. Let $ax + b = 15$ and $15x + a = b$ have the same unique solution, where a and b are positive integers both less than or equal to 30. What is the sum of all possible values of a ?

- a. 28 b. 43 c. 58 d. 78 e. 93

16. Different letters are placed on the 18 faces of 3 standard 6-sided dice, one per face. Choosing 1 letter from each die, the following words can be formed: bow, boy, cot, dry, gas, hat, oat, old, one, pay, pie, red, six. Which of the following could also be spelled?

- a. won b. rap c. eat d. wad e. top

17. There is enough food in a pig pen to feed 14 pigs for 16 days. For how many days will this amount feed 8 pigs?
- a. 20 days b. 18 days c. 28 days d. 14 days e. 21 days
18. If $\log_3 10 = 2.095903$, then how many digits does 100,000 have when expressed in base 3?
- a. 12 b. 13 c. 10 d. 11 e. 8
19. An isosceles triangle has equal sides of length A and the altitude to the third side is length B . Which of the following represents the radius of the circumscribed circle of the triangle?
- a. $\frac{A^2}{2B}$ b. $\frac{B^2}{4A}$ c. $\frac{B\sqrt{2}}{2A}$ d. $\frac{A^2}{4B}$ e. $\frac{B^2}{2A}$
20. The vertices of a square in the first quadrant are $(x, 0)$, $(0, y)$, (a, b) , and (c, d) . What is $x + y$, if $a + b = 19$ and $c + d = 14$?
- a. 17 b. 9 c. 11 d. 15 e. 13
21. If $x + \sqrt{x^2 - 1} + \frac{1}{x + \sqrt{x^2 - 1}} = 20$, then what is $x^2 + \sqrt{x^4 + 1} + \frac{1}{x^2 - \sqrt{x^4 + 1}}$?
- a. 0 b. 21 c. 51.005 d. 61.25 e. 200

22. Mary and Jackie are “empty-nesters” (that means no children living at home). They decide that they should go out to dinner three nights a week for the next year. To keep things interesting they stipulate that they will not go to the same restaurant more than once in any week AND no one week may contain the same three restaurants as any other week. What is the minimum number of restaurants that make this possible?
- a. 20 b. 8 c. 12 d. 10 e. 6
23. An infinite geometric series has a common ratio $\frac{2}{3}$ and sums to the value S . If the first, third, and all other odd terms are doubled, and the second, fourth, and all other even terms are halved, what is the sum of the new series?
- a. $40.6S$ b. $0.8S$ c. $1.0S$ d. $1.2S$ e. $1.4S$
24. If $\{r, s, t, u, v\}$ satisfies the system
$$\begin{cases} 3r + 10s + 16t + 30u + 25v = 10 \\ 4r + 15s + 20t + 36u + 36v = 11 \\ 5r + 20s + 24t + 42u + 49v = 20 \end{cases}$$
, then what is the value of $6r + 25s + 28t + 48u + 64v$?
- a. 34 b. 33 c. 37 d. 36 e. 35
25. Five murder suspects, including the guilty party, are being interrogated by the police. Results of a polygraph indicate two of them are lying and three are telling the truth. If the polygraph results are correct, who committed the murder?
- Suspect A: “Suspect D is the murderer.”
- Suspect B: “I am innocent.”
- Suspect C: “It wasn’t Suspect E.”
- Suspect D: “Suspect A is lying.”
- Suspect E: “Suspect B is telling the truth.”
- a. B b. E c. A d. C e. D

SHORT ANSWER

Place the answer in the appropriate space.

66. Determine the four digit perfect square of the form $xyyy$, where x and y are integers and $x \neq 0$.

67. What is the smallest integer n for which $n!$ terminates in exactly 66 zeros?

68. Let n be an integer such that $\sqrt[3]{n + \sqrt{n^2 + 8}} + \sqrt[3]{n - \sqrt{n^2 + 8}} = 8$. What is the value of n ?

69. The sum of two roots of a fourth-degree polynomial function with integer coefficients is $4 + \sqrt{3}$, and the product of the other two roots is $-6 + 3\sqrt{3}$. If two of the four roots are integers, then what is the sum of the coefficients of the polynomial?

70. If $n \neq 0$, then what does $\sqrt[n]{\frac{2^{2n+4} + 2^{2n+2}}{20}}$ equal?

Answer Key

1. d
2. a
3. d
4. b
5. d
6. e
7. c
8. e
9. e
10. b
11. c
12. a
13. e
14. d
15. c
16. a
17. c
18. d
19. a
20. c
21. a
22. b
23. e
24. c
25. b

66. 7744
67. 270
68. 280
69. 8
70. 4