

2016 State Math Contest Level II Test

1. A certain disease occurs in 20% of the male population and the test for it is 90% accurate (which means 90% of the time the test correctly identifies who does or who does not have the disease). If a man tests positive which of the following is closest to the chance that he truly has the disease?
a. 80% b. 70% c. 60% d. 50% e. 20%

2. Amongst the children in a family a boy has as many sisters as brothers, but each sister has only half as many sisters as brothers. How many children are in the family?
a. 7 b. 5 c. 6 d. 4 e. 9

3. A translation takes the point $(2, -5)$ and moves it to the point $(-6, 4)$. If the image of another point under the same translation is $(-1, 8)$, what are the coordinates of its pre-image?
a. $(-2, 7)$ b. $(4, -6)$ c. $(-2, 8)$ d. $(3, -4)$ e. $(7, -1)$

4. Three of the vertices of rectangle $ABCD$ are $A(-3, 1)$, $B(1, 1)$, and $C(1, 3)$. The rectangle is translated 3 units to the right and 7 units down. The resulting rectangle $A'B'C'D'$ is then rotated clockwise by 90° about B' . What are the coordinates of the image of D after the two transformations?
a. $(6, -2)$ b. $(6, -1)$ c. $(-3, -2)$ d. $(5, -3)$ e. $(4, -1)$

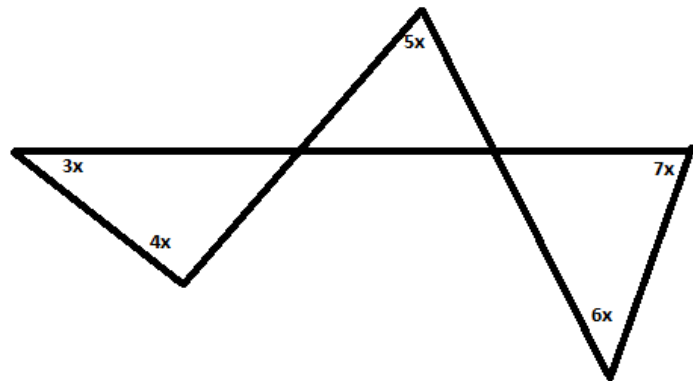
5. A pharmacist has an order for 20 gallons of 80% pure alcohol. (Distilled water accounts for the other 20%.) She only has 65% and 90% pure alcohol in stock, but she is able to fill the order. How much more of the 90% solution does she need to use than the 65% solution?
a. 2 gallons b. 3.5 gallons c. 4 gallons d. 4.5 gallons e. 6 gallons

6. How many of the following seven shapes has at least one line of symmetry: rectangle, parallelogram, trapezoid, regular pentagon, triangle, circle, and hexagon?
- a. 6 b. 5 c. 4 d. 3 e. 2
7. Michael's tires are 18 inches in diameter. There is a nail in his tire and he drives 5 miles (assume the tire does not go flat in the drive). Which of the following most closely approximates the number of times the nail touches the road in the 5-mile drive?
- a. 6000 b. 5500 c. 5600 d. 5900 e. 6200
8. Given $f(x) = 2x^2 - 4$, what is the equation of the line determined by the point $P(1, f(1))$ and $Q(5, f(5))$?
- a. $y = 12x - 10$ b. $y = 12x + 10$ c. $y = 12x - 14$ d. $y = 11x - 13$ e. $y = 11x - 9$
9. Define a "micro-century" as one millionth of a century. Which of the following most closely approximates the length of a "micro-century"?
- a. 5 seconds b. 50 minutes c. 5 hours d. 50 seconds e. 5 minutes
10. The total mass of a bottle and its contents of 20 identical tablets was 180 grams. When the bottle contained 15 tablets the total mass was 165 grams. What is the total mass of the bottle?
- a. 103 g b. 115 g c. 120g d. 125 g e. 146 g

11. The vertices of $\triangle ABC$ are $A(0,12)$, $B(9,0)$, and $C(0,0)$. The vertices of $\triangle A'B'C'$ are $A'(12,0)$, $B'(0,0)$, and $C'\left(\frac{27}{5}, \frac{36}{5}\right)$. What is the area of the region that lies in both of the two triangles?

a. 54 b. 27 c. $9\sqrt{3}$ d. $8\sqrt{3}$ e. 36

12. In the diagram the $3x$, $4x$, $5x$, $7x$, $6x$ are the measures of the angles in degrees. What is the value of x ?



a. 6 b. 8 c. 10 d. 12 e. 15

13. Three faces of a rectangular box have a common point, which is a corner of the box. The centers of these faces are the vertices of a triangle with sides of length 4, 5, and 6 cm. What is the volume of the box?

a. $120\sqrt{2} \text{ cm}^3$ b. $45\sqrt{3} \text{ cm}^3$ c. $45\sqrt{6} \text{ cm}^3$ d. 125 cm^3 e. $90\sqrt{6} \text{ cm}^3$

14. A circular sporting field has fencing stretching around the boundary. How much longer in feet would the fence need to be increased to increase the radius of the playing field by 30 feet?

a. $30\pi \text{ ft}$ b. $15\pi \text{ ft}$ c. 30 ft d. $60\pi \text{ ft}$ e. 60 ft

15. What is the area of the set of points (x, y) defined by $|2x - 3y| \leq 12$ and $|2x + 3y| \leq 12$?
- a. 24 b. 144 c. 72 d. 48 e. 96
16. The vertices of a square in the first quadrant are $(x, 0)$, $(0, y)$, (a, b) , and (c, d) . What is $x + y$, if $a + b = 19$ and $c + d = 14$?
- a. 11 b. 9 c. 17 d. 15 e. 13
17. The sides of a triangle are 5, 6, and 7. What is the ratio of the area of the triangle to its perimeter?
- a. $\frac{\sqrt{6}}{3}$ b. $\sqrt{6}$ c. $\frac{2\sqrt{3}}{3}$ d. $\frac{\sqrt{6}}{2}$ e. $\frac{3\sqrt{3}}{2}$
18. What is the distance between the two centers of the circles with equations $x^2 + y^2 - 8x - 8y + 16 = 0$ and $x^2 + y^2 + 2x + 16y + 9 = 0$?
- a. $2\sqrt{10}$ b. 13 c. 15 d. $4\sqrt{6}$ e. 5
19. There is enough food in a pig pen to feed 14 pigs for 16 days. For how many days will this amount feed 8 pigs?
- a. 20 days b. 18 days c. 21 days d. 14 days e. 28 days

20. In rectangle $ABCD$, $AB = 6$ and $BC = 2$. If E is between A and B such that $AE = BC$ and F is the intersection of lines BD and CE , what is the area of $\triangle BCF$?
- a. 2.2 b. 2.4 c. 2.6 d. 2.8 e. 3.0
21. It takes Michaela 3.5 hours to prepare a large flower delivery for sale as small bouquets. When Nikhil helps her, the two complete the job in two hours. If Nikhil had to do the job by himself, how much time would he need?
- a. 4.5 hours b. $\frac{11}{3}$ hours c. 5 hours d. $\frac{14}{3}$ hours e. 4.8 hours
22. The numbers one through twelve are written in two rows and six columns so that the 2 row sums are equal to one another, and the 6 column-sums are also equal to one another. If the number eight appears in the first row, then how many even numbers are there in the second row?
- a. 1 b. 2 c. 3 d. 4 e. 5
23. Mabel throws two fair, six-sided dice, a red one and a white one. What is the probability that the red one beats the white one?
- a. $\frac{1}{2}$ b. $\frac{1}{6}$ c. $\frac{5}{12}$ d. $\frac{7}{12}$ e. $\frac{1}{3}$
24. An isosceles triangle has equal sides of length A and the altitude to the third side is length B . Which of the following represents the radius of the circumscribed circle of the triangle?
- a. $\frac{B^2}{2A}$ b. $\frac{B^2}{4A}$ c. $\frac{B\sqrt{2}}{2A}$ d. $\frac{A^2}{4B}$ e. $\frac{A^2}{2B}$

25. Five murder suspects, including the guilty party, are being interrogated by the police. Results of a polygraph indicate two of them are lying and three are telling the truth. If the polygraph results are correct, who committed the murder?

Suspect A: "Suspect D is the murderer."

Suspect B: "I am innocent."

Suspect C: "It wasn't Suspect E."

Suspect D: "Suspect A is lying."

Suspect E: "Suspect B is telling the truth."

- a. E b. D c. C d. B e. A

SHORT ANSWER

Place the answer in the appropriate space.

66. If a and b are integers such that $a^2 + b^2 = 29$, what is the maximum value of $|a - b|$?
67. Circle O has radius 10 and is tangent to $\angle BAC$ at points B and C . Circle Q is tangent to circle O at point P and to $\angle BAC$ at points D and E . Circle S is tangent to circle Q at point R and to $\angle BAC$ at points F and G . If $OP > QR > RS$ and $OA = 25$, what is $\sqrt{SA}$?
68. Two rectangles of the same dimensions $ABDE$ and $ACDF$ have a common diagonal $\overline{AD}$. What is the ratio of the longest side of the rectangles to the shorter side of the rectangles such that $ABCDEF$ forms a regular hexagon?
69. Let A and B be endpoints of a diameter of a circle. Point C is located on the circle such that the ratio of the area of the circle to the area of $\triangle ABC$ is 2π . What is the measure of the smallest angle of the triangle in degrees?
70. The consecutive sides of a trapezoid have lengths 6, 13, 27, and 20. What is the area of the trapezoid?

Answer Key

1. b
2. a
3. e
4. a
5. c
6. d
7. c
8. c
9. b
10. c
11. b
12. d
13. e
14. d
15. d
16. a
17. a
18. b
19. e
20. b
21. d
22. e
23. c
24. e
25. a

66. 7

67. $\frac{15}{7}$

68. $\sqrt{3}$

69. 15°

70. 198