

2016 State Math Contest Level I Test

1. A certain disease occurs in 20% of the male population and the test for it is 80% accurate (which means 80% of the time the test correctly identifies who does or who does not have the disease). If a man tests positive which of the following is closest to the chance that he truly has the disease?
a. 80% b. 70% c. 60% d. 50% e. 20%

2. A store advertises, "We pay the sales tax!" If sales tax is 8%, what discount to the buyer to the nearest tenth of a percent does this represent?
a. 7.1% b. 7.2% c. 7.4% d. 7.5% e. 7.8%

3. The two distinct lines with equations $ax + y = c$ and $x + y = c$ intersect at the point $(b - 2, 4)$. What is $b + c$?
a. 8 b. 6 c. 4 d. 2 e. 0

4. Margaret has an equal number of dimes and nickels in her pocket and only four pennies. After spending 15 cents, she has twice as many dimes as nickels. How many nickels did she have originally?
a. 3 b. 7 c. 5 d. 4 e. 6

5. The students in an 8th grade math class in a small school (less than 80 students per grade) can be seated in rows of four or three, each time with exactly the same number of seats in each row. When seated in rows of five, one row has exactly 1 fewer students than all the other rows. If four new students join the class, in how many equal rows could her students now be seated?
a. 7 b. 10 c. 9 d. 8 e. 6

6. A translation takes the point $(2, -5)$ and moves it to the point $(-6, 4)$. If the image of another point under the same translation is $(-1, 8)$, what are the coordinates of its pre-image?
a. $(7, -1)$ b. $(4, -6)$ c. $(-2, 8)$ d. $(3, -4)$ e. $(-2, 7)$

7. Two fair six-sided dice are tossed. If at least one of the dice shows the number four, what is the probability that the sum of the two dice is greater than eight?
- a. $\frac{5}{12}$ b. $\frac{2}{11}$ c. $\frac{4}{11}$ d. $\frac{1}{3}$ e. $\frac{5}{11}$
8. Determine how many integers satisfy the following condition: When twice the integer is subtracted from twice the square of the integer, the result is at most twelve.
- a. 0 b. 6 c. 8 d. 4 e. 10
9. Define a “micro-century” as one millionth of a century. Which of the following most closely approximates the length of a “micro-century”?
- a. 5 seconds b. 50 minutes c. 5 hours d. 50 seconds e. 5 minutes
10. Amongst the children in a family a boy has as many sisters as brothers, but each girl has only half as many sisters as brothers. How many children are in the family?
- a. 7 b. 5 c. 6 d. 4 e. 9
11. How many of the following seven shapes has at least one line of symmetry: rectangle, parallelogram, trapezoid, regular pentagon, triangle, circle, and hexagon?
- a. 2 b. 3 c. 4 d. 5 e. 6
12. Let $f(x) = x^2$. Define $P(a, b)$ to be the y -coordinate of the y -intercept of the line through $(a, f(a))$ and $(-b, f(-b))$. Determine $P(4, 6) + P(3, 5)$.
- a. 30 b. 33 c. 36 d. 39 e. 42

13. Mabel throws two fair, six-sided dice, a red one and a white one. What is the probability that the red one beats the white one?
- a. $\frac{1}{2}$ b. $\frac{1}{6}$ c. $\frac{5}{12}$ d. $\frac{7}{12}$ e. $\frac{1}{3}$
14. It takes Michaela 3.5 hours to prepare a large flower delivery for sale as small bouquets. When Nikhil helps her, the two complete the job in two hours. If Nikhil had to do the job by himself, how much time would he need?
- a. 4.5 hours b. $\frac{11}{3}$ hours c. 5 hours d. $\frac{14}{3}$ hours e. 4.8 hours
15. The product $\sqrt[5]{16}\sqrt[3]{4}$ can be expressed as $2^{\frac{p}{q}}$, where $\frac{p}{q}$ is a reduced fraction. What is $p+q$?
- a. 28 b. 37 c. 15 d. 45 e. 34
16. What is the sum of the solutions of $3\sqrt{2-x} = x-4$?
- a. 1 b. 2 c. -1 d. -2 e. no solutions
17. A fraction is chosen at random from all positive unreduced proper fractions with denominators less than 6. What is the probability that the fraction's decimal representation terminates?
- a. $\frac{4}{5}$ b. $\frac{2}{3}$ c. $\frac{7}{10}$ d. $\frac{7}{9}$ e. $\frac{3}{5}$

18. If a and b are real numbers such that $0 < a < b$ and $a^2 + b^2 = 6ab$, then what is the value of $\frac{a+b}{a-b}$?
- a. -2 b. -1 c. 0 d. $-\sqrt{2}$ e. $\sqrt{2}$
19. A pharmacist has an order for 20 gallons of 80% pure alcohol. (Distilled water accounts for the other 20%.) She only has 65% and 90% pure alcohol in stock, but she is able to fill the order. How much more of the 90% solution does she need to use than the 65% solution?
- a. 2 gallons b. 3.5 gallons c. 4 gallons d. 4.5 gallons e. 6 gallons
20. The roots of the equation $x^3 - 18x^2 + 107x - d = 0$ are distinct consecutive integers. What is the value of d ?
- a. 720 b. 120 c. 336 d. 240 e. 210
21. The numbers one through twelve are written in two rows and six columns so that the 2 row sums are equal to one another, and the 6 column-sums are also equal to one another. If the number eight appears in the first row, then how many even numbers are there in the second row?
- a. 1 b. 2 c. 3 d. 4 e. 5
22. Given $f(x) = 2x^2 - 4$, what is the equation of the line determined by the point $P(1, f(1))$ and $Q(5, f(5))$?
- a. $y = 12x - 10$ b. $y = 12x + 10$ c. $y = 12x - 14$ d. $y = 11x - 13$ e. $y = 11x - 9$

23. The total mass of a bottle and its contents of 20 identical tablets was 180 grams. When the bottle contained 15 tablets the total mass was 165 grams. What is the total mass of the bottle?
- a. 103 g b. 115 g c. 120g d. 125 g e. 146 g
24. A collection of 41 coins is worth exactly \$1.00. If it has at least one quarter, one dime, one nickel, and one penny, what is the total number of dimes and nickels?
- a. 5 b. 6 c. 7 d. 8 e. 9
25. Five murder suspects, including the guilty party, are being interrogated by the police. Results of a polygraph indicate two of them are lying and three are telling the truth. If the polygraph results are correct, who committed the murder?
- Suspect A: "Suspect D is the murderer."
- Suspect B: "I am innocent."
- Suspect C: "It wasn't Suspect E."
- Suspect D: "Suspect A is lying."
- Suspect E: "Suspect B is telling the truth."
- a. A b. B c. C d. D e. E

SHORT ANSWER

Place the answer in the appropriate space.

66. How many zeros are there in a septillion?
67. Will jogs from home to school at 9 mph, then runs from school to the store at 4.5 mph, and finally bikes back home. Home, school, and the store are equidistant from each other. If Will's average speed is 7.5 mph, then what is his biking speed?
68. Each member of a group of five people is either a squint, who always tells the truth, or a squat, who always lies. Each member of the group looks at everyone else in the group and then one member says, "I see at least one person who sees only squats." What is the least number of squints in the group?
69. A domino set has twenty eight pieces (part of the set pictured to the right). Ignoring the double blank the remaining 27 pieces may be viewed as fractions less than or equal to one. When summed what is their total?
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70. In my town some of the animals are really strange. Ten percent of the dogs think they are cats and ten percent of the cats think they are dogs. All other cats and dogs are perfectly normal. One day I tested all the cats and dogs in the town and found that 20% of them thought that they were cats. What percentage of the animals I tested really were cats?

Answer Key

1. d
2. c
3. b
4. e
5. a
6. a
7. d
8. b
9. b
10. a
11. b
12. d
13. c
14. d
15. b
16. e
17. a
18. d
19. c
20. e
21. e
22. c
23. c
24. a
25. e

66. 24
67. 15
68. 2
69. 13.5
70. 12.5%