Level III
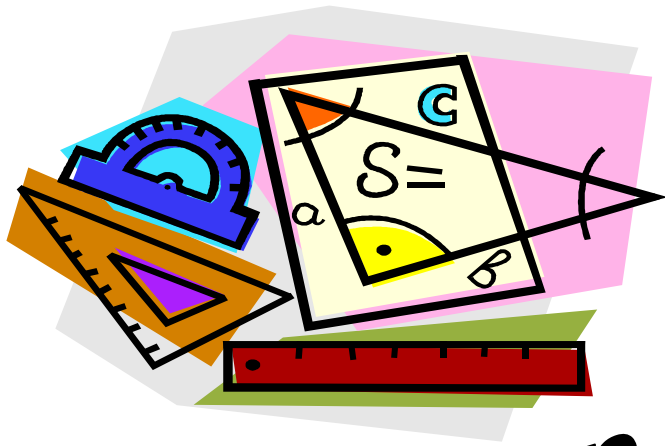
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April 9, 2015

1. According to a survey conducted by Parade magazine, the average American ate 405 savory snacks, 366 sweet snacks, and 357 healthier snacks in 2013. Since 2006, healthier snacks are up 14% while sweet snacks are down 6%. Using this data which of the following would best represent the number of sweet snacks consumed by the average American in 2006?

    a. 405   b. 392   c. 375   d. 388   e. 389

2. If n is an integer, which of the following must be an even integer?

    a. $5n-1$ b. n^2+5n+6 c. n^3 d. $n+6$ e. n^2+4n+4

3. If $f(x)=|x|+|x-1|+|x+1|$ then what is the minimum value of $f(x)$?

    a. 1   b. 3   c. 4   d. 2   e. 0

4. Which of the following is a possible value of $x^{3y}+2$ if $x^{2y}=16$?

    a. 18   b. 66   c. 6   d. 34   e. 130

5. How many different ordered pairs satisfy both $x^2+y^2=100$ and $x^2+2y^2=108$?

    a. 4   b. 3   c. 2   d. 1   e. 0

6. A book is to have 250 pages that will be numbered with Arabic numerals beginning with 1. How many times will the digit 2 be used in numbering the pages?

    a. 107   b. 106   c. 105   d. 56   e. 85

7. Let m and n be integers. How many solutions (m,n) of the inequality $4\le m^2+n^2\le17$ are there?

    a. 48   b. 36   c. 44   d. 50   e. 52

8. Four teachers bought an old boat for $60. The first teacher paid one-half of the sum of the amounts paid by the other teachers; the second teacher paid one-third of the sum of the amounts paid by the other teachers; the third teacher paid one-fourth of the sum of the amounts paid by the other teachers. How much did the fourth teacher pay?

    a. $10   b. $12   c. $13   d. $14   e. $15

9. Betty likes to drink a mixture of fruit juice and lemonade. One day she filled a glass half full with fruit juice, then filled it up with lemonade. After mixing the two liquids thoroughly, she drank one third of the amount, and again filled the glass with lemonade. What fraction of the final mixture was fruit juice?

    a. $\dfrac{1}{6}$ b. $\dfrac{1}{3}$ c. $\dfrac{1}{2}$ d. $\dfrac{1}{4}$ e. $\dfrac{2}{3}$

10. Which of the following does $\left(1+x^3\right)\left(1-x^3+x^6\right)$ equal?

    a. $1-x^9$ b. $1+x^3-x^6+x^9$ c. $1-x^6+x^9$ d. $1-2x^6+x^9$ e. $1+x^9$

11. What is the greatest whole number that is a factor of the sum of any four consecutive positive even integers?

    a. 16   b. 8   c. 4   d. 2   e. 1

12. Let 5, 9, and c be the lengths of the sides of a triangle. If c is an integer, then what is the difference between the largest and smallest possible value of c?

    a. 6   b. 7   c. 9   d. 8   e. 10

13. What is the maximum number of obtuse angles that any convex polygon can have?

    a. 50   b. 100   c. 360   d. 1000   e. no limit

14. An equilateral triangle and a regular hexagon have the same perimeter. What is the ratio of the area of the hexagon to the area of the triangle?

   a. 1.5   b. 2   c. $\dfrac{3\sqrt{3}}{2}$ d. $\dfrac{4\sqrt{3}}{3}$ e. 1

15. During a windstorm, a 32-foot vertical flag pole positioned on flat ground cracks in such a way that the top of the pole touches the ground 12 feet from the base of the pole. How many feet above the ground did the break occur?

   a. 12.5 ft   b. 13 ft   c. 12.75 ft   d. 13.75 ft   e. 13.5 ft

16. Two 8-sided dice each have faces numbered 1 through 8. When the dice are rolled, each number has an equal probability of appearing on the top. What is the probability that the product of the two numbers on the top of the dice is greater than their sum?

   a. $\dfrac{3}{4}$ b. $\dfrac{3}{8}$ c. $\dfrac{1}{2}$ d. $\dfrac{9}{16}$ e. $\dfrac{45}{64}$

17. Suppose that $f(x)=ax+b$, where a and b are real numbers. If $f(f(f(x)))=8x+21$, what is the value of $a+b$?

   a. 4   b. 6   c. 13   d. 29   e. 5

18. If $xy=6$ and $x^2y+xy^2+x+y=63$, then what is the value of x^2+y^2?

   a. 36   b. 69   c. 48   d. 54   e. 60

19. A circle of radius 4 is centered at the origin. Its radius increases 3 units every second. Another circle of radius 12 is centered at $(30,0)$. Its radius decreases 1 unit every second. When the circles meet, where will the point $(27,4)$ lie?

    a. in the first circle   b. on the first circle   c. in the second circle

                d. on the second circle   e. between the circles

20. A print shop takes 2.4 hours to run a job on the P10 and P5 copiers. The same job takes 3 hours on the P10 and P2 copiers, and four hours on the P5 and P2 copiers. How many hours would the job take running on all three machines?

    a. 1.2   b. 1.5   c. 1.6   d. 1.8   e. 2.0

21. How many irrational solutions does the equation $18x^4 - 11x^2 + 1 = 0$ have?

    a. 0   b. 1   c. 2   d. 3   e. 4

22. Determine the number of different arrangements that can be made from the letters in the word CONTESTS.

    a. 720   b. 40,320   c. 10,080   d. 20,160   e. 5,040

23. The model for the amount of a decaying isotope, A, in grams, is given by $A(t) = 20(0.98)^t$, where t is the number of days since 20 grams were present. Determine the half-life of the isotope A.

    a. $\ln(0.5)$ b. $\ln(0.49)$ c. $\ln(0.98)$ d. $\log_{0.98}(0.5)$ e. $\log_{0.5}(0.98)$

24. Which of the following intervals is a subset of the domain of $f(x) = \sqrt{\dfrac{x - \dfrac{3}{x-4}}{\dfrac{x+2}{x-2}}}$?

   a. $(-2, -0.5)$ b. $(2, 5)$ c. $(5, \infty)$ d. $(-\infty, -2)$ e. none of these

25. The top three finishers in a recent math contest were Greg, Tonya , and Stephen. These three students attended three different high schools named North, West, and Central. The student from Central High School finished in second place. Two times Greg's score minus the score of the student from West HS equaled the score of the first place finisher. Tonya scored a 106 on the contest, but did not finish in first place. The difference between first place and second place was four points. What was the sum of the top three scores?

   a. 330   b. 324   c. 322   d. 320   e. 336

SHORT ANSWER
Place the answer in the appropriate space.

66. For the function $f(x)$, $f(1)=3$ and $f(x)+f(y)=f\left(\dfrac{x+y}{2}\right)f\left(\dfrac{x-y}{2}\right)$ for all real numbers x and

y .Calculate the value of $f(3)$.

67. Determine the value of b such that the system below has an infinite number of solutions.
$$-3x+2y-4z=6$$
$$7x+6y+4z=b$$
$$-5x-4y-3z=-1$$

68. The length of the sides of a triangle are the roots of the equation $x^3-16x^2+85x-150=0$. What is the area of the triangle?

69. Let $\log_a b+\log_b a=\sqrt{29}$, then what is $\left|\log_a b-\log_b a\right|$?

70. What is the sum of the solutions to the equation $3\left(3^{2x}\right)-28\left(3^x\right)=-9$?

Answer Key

1. e
2. b
3. d
4. b
5. a
6. b
7. a
8. c
9. b
10. e
11. c
12. d
13. e
14. a
15. d
16. a
17. e
18. b
19. d
20. e
21. c
22. c
23. d
24. c
25. a

66. 18
67. 2
68. 12
69. 5
70. 1