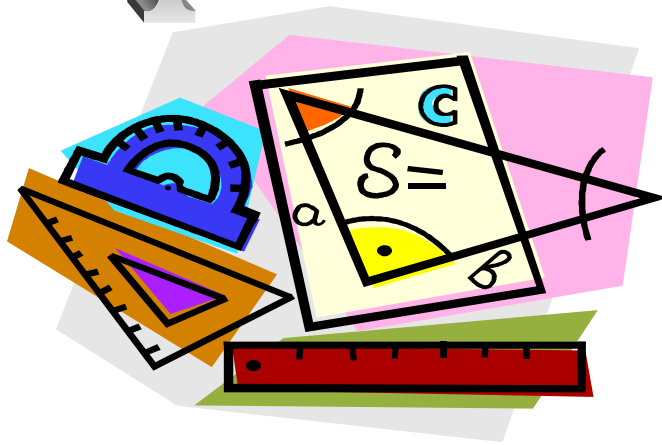


# Comprehensive



**Do NOT open until  
you are told to do so.**

April 9, 2015

1. According to a survey conducted by Parade magazine, the average American ate 405 savory snacks, 366 sweet snacks, and 357 healthier snacks in 2013. Since 2006, healthier snacks are up 14% while sweet snacks are down 6%. Using this data which of the following would best represent the number of sweet snacks consumed by the average American in 2006?  
a. 385                      b. 389                      c. 388                      d. 387                      e. 390
  
2. Given that  $\begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} x & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ y & -10 \end{bmatrix}$ , determine  $5x+3y$ .  
a. 39                      b. 25                      c. 11                      d. 44                      e. 49
  
3. Two runners running around a 600 m track in opposite directions and starting from the same place run a lap in 100 sec and 150 sec respectively. How many meters from the starting place are the runners when they meet for the eighth time (not counting the start)?  
a. 60 m                      b. 120 m                      c. 180 m                      d. 240 m                      e. 300 m
  
4. A streetlight and a nearby building simultaneously cast shadows of lengths 52 ft and 88ft respectively. If the streetlight is known to be 39 ft high, how tall is the building?  
a. 60 ft                      b. 66 ft                      c. 70 ft                      d. 75 ft                      e. 78 ft
  
5. Determine the number of different arrangements that can be made from the letters in the word CONTESTS.  
a. 720                      b. 40,320                      c. 10,080                      d. 20,160                      e. 5,040

6. The model for the amount of a decaying isotope,  $A$ , in grams, is given by  $A(t) = 20(0.98)^t$ , where  $t$  is the number of days since 20 grams were present. Determine the half-life of the isotope  $A$ .

- a.  $\ln(0.5)$       b.  $\ln(0.49)$       c.  $\ln(0.98)$       d.  $\log_{0.5}(0.98)$       e.  $\log_{0.98}(0.5)$

7. Determine the distance between the focus of the parabola  $r = \frac{10}{2 + 2 \cos \theta}$  and the center of the circle  $r = 10 \cos(\theta)$ .

- a. 2      b. 2.5      c. 5      d.  $5\sqrt{2}$       e. 10

8. Given  $f(x) = \frac{2x^2 - 4x - 6}{9 - x^2}$ , determine which statements below are true.

- i)  $f(x)$  has a horizontal asymptote of  $y = -2$ .
- ii)  $f(x)$  has vertical asymptotes of  $x = 3$  and  $x = -3$ .
- iii)  $f(3) = \frac{4}{3}$

- a. i only      b. ii only      c. i and ii      d. all of them      e. none of them

9. Given the numerical functions  $f$  and  $g$  below, determine which statements are true.

- i)  $f^{-1}(4) = 5$
- ii)  $f(g(8)) = 5$
- iii)  $2\sqrt{(g^{-1}(8))} = 4$

$x$	$f(x)$	$g(x)$
2	4	10
4	5	8
6	6	6
8	7	4
10	8	2

- a. ii only      b. iii only      c. i and ii      d. ii and iii      e. all of them

10. If  $f(x)$  is a cubic polynomial with real coefficients such that  $f(2) = 0$ ,  $f(1+i) = 0$ , and  $f(0) = 2$ , what is  $f(-2)$ ?

- a. 20                      b. 40                      c. -4                      d. -40                      e. -20

11. Which of the following intervals is a subset of the domain of  $f(x) = \sqrt{\frac{x-3}{\frac{x+2}{x-2}}}$  ?

- a.  $(-2, -0.5)$                       b.  $(2, 5)$                       c.  $(5, \infty)$                       d.  $(-\infty, -2)$                       e. none of these

12. What is the greatest whole number that is a factor of the sum of any four consecutive positive even integers?

- a. 16                      b. 8                      c. 4                      d. 2                      e. 1

13. Let 5, 9, and  $c$  be the lengths of the sides of a triangle. If  $c$  is an integer, then what is the difference between the largest and smallest possible value of  $c$ ?

- a. 4                      b. 5                      c. 6                      d. 7                      e. 8

14. What is the maximum number of obtuse angles that any convex polygon can have?

- a. 50                      b. 100                      c. 360                      d. 1000                      e. no limit

15. Given that  $\frac{x}{y} = 2^z$ ,  $\frac{y^2}{z} = x$ , and  $\frac{yz}{\sqrt{x}} = 8$ , determine  $\log_2(xyz)$ .

- a. 10                      b. 16                      c. 8                      d. 18                      e. 20

16. Two 8-sided dice each have faces numbered 1 through 8. When the dice are rolled, each number has an equal probability of appearing on the top. What is the probability that the product of the two numbers on the top of the dice is greater than their sum?
- a.  $\frac{3}{4}$                       b.  $\frac{3}{8}$                       c.  $\frac{1}{2}$                       d.  $\frac{9}{16}$                       e.  $\frac{45}{64}$
17. Mrs. Abbott finds that the number of possible groups of three students in her class is exactly five times the number of possible groups of two students. How many students are in her class?
- a. 15                      b. 25                      c. 19                      d. 22                      e. 17
18. A circle of radius 4 is centered at the origin. Its radius increases 3 units every second. Another circle of radius 12 is centered at  $(30,0)$ . Its radius decreases 1 unit every second. When the circles meet, where will the point  $(27,4)$  lie?
- a. in the first circle                      b. on the first circle                      c. in the second circle  
d. on the second circle                      e. between the circles
19. A print shop takes 2.4 hours to run a job on the HP10 and HP5 copiers. The same job takes 3 hours on the HP10 and HP2 copiers, and four hours on the HP5 and HP2 copiers. How many hours would the job take running on all three machines?
- a. 1.2                      b. 1.5                      c. 1.6                      d. 1.8                      e. 2.0
20. How many irrational solutions does the equation  $18x^4 - 11x^2 + 1 = 0$  have?
- a. 0                      b. 1                      c. 2                      d. 3                      e. 4

21. A triangle has sides of length  $a < b < c$ , which are consecutive integers and  $\cos(C) = \frac{5}{16}$ . Determine  $\cos(A)$ .
- a.  $\frac{13}{20}$                       b.  $\frac{7}{11}$                       c.  $\frac{5}{8}$                       d.  $\frac{2}{3}$                       e.  $\frac{11}{16}$
22. What is the maximum number of pieces into which a circular pizza can be cut by making 8 straight cuts?
- a. 36                      b. 28                      c. 16                      d. 37                      e. 40
23. A right triangle has legs  $6\sqrt{13}$  and  $4\sqrt{13}$ . What is the distance between the midpoint of the hypotenuse and the foot of the altitude to the hypotenuse?
- a. 5                      b. 8                      c.  $\frac{\sqrt{13}}{2}$                       d.  $\sqrt{13}$                       e. 4
24. How many zeroes are needed to write the base-10 number  $2^{2^{2^2}}$  in base 2?
- a. 10                      b. 16                      c. 12                      d. 11                      e. 14
25. The top three finishers in a recent math contest were Greg, Tonya, and Stephen. These three students attended three different high schools named North, West, and Central. The student from Central High School finished in second place. Two times Greg's score minus the score of the student from West HS equaled the score of the first place finisher. Tonya scored a 106 on the contest, but did not finish in first place. The difference between first place and second place was four points. What was the sum of the top three scores?
- a. 322                      b. 336                      c. 320                      d. 330                      e. 324

**SHORT ANSWER**

Place the answer in the appropriate space.

66. To determine divisibility by 7 you subtract twice the last digit from the number formed by removing the last digit. If the difference is divisible by seven, then the original number is divisible by 7. For example 161 is divisible by 7 because  $16 - 2 \cdot 1 = 14$  is divisible by 7. If you subtract  $k$  times the last digit (instead of twice), for what value of  $k$  does this determine divisibility by 13?

67. For the function  $f(x)$ ,  $f(1) = 3$  and  $f(x) + f(y) = f\left(\frac{x+y}{2}\right) + f\left(\frac{x-y}{2}\right)$  for all real numbers  $x$  and  $y$ . Calculate the value of  $f(5)$ .

68. Given the points  $A(0,0)$ ,  $B(2,8)$ ,  $C(6,2)$ , and  $D(4,4)$ , determine the area of the region in the first quadrant consisting of those points closer to A than to any of the other 3 points.

69. You have eight white socks which are identical except for the color of the toe. In how many ways can you separate the socks into four different pairs?

70. A package of Plenty and Good has candies colored red and white. Every box has between 30 and 40 candies. In a particular box of Plenty and Good, Ted noticed that when two candies were randomly selected from the box, the probability that the two candies are the same color is one-half. How many candies are in this particular box of Plenty and Good?

**Answer Key**

1. b
2. a
3. b
4. b
5. c
6. e
7. c
8. a
9. d
10. a
11. c
12. c
13. e
14. e
15. d
16. a
17. e
18. d
19. e
20. c
21. a
22. d
23. a
24. b
25. d

66. 9

67. 123

68.  $\frac{23}{3}$

69. 105

70. 36