

1. The Bluebird zip line starts 42 feet above the ground and ends 6 feet above the ground. The horizontal distance covered by the zip line is 60 yards. Which of the following is the slope of the Bluebird zip line?
a. 0.7 ft per yd b. 0.2 in per in c. 0.6 yd per yd d. 0.6 ft e. 0.2 yd

2. All of the coefficients of the fourth degree polynomial $P(x)$ are odd integers. What is the maximum number of rational solutions of the equation $P(x) = 0$?
a. 0 b. 1 c. 2 d. 3 e. 4

3. Solve the equation: $12\log 12 = 3x\log 3 + 2x\log 8$
a. 12 b. 8 c. 6 d. 2 e. 4

4. Right triangle ABC has a right angle at C . M is the midpoint of \overline{AB} . If $BC = 7$ in and $MC = 12.5$ in, what is $\sin(\angle CMB)$?
a. $\frac{24}{25}$ b. $\frac{7}{25}$ c. $\frac{168}{625}$ d. $\frac{336}{625}$ e. $\frac{7}{48}$

5. For how many three-digit integers between 100 and 999 do the first and last digits differ by four?
a. 110 b. 130 c. 140 d. 120 e. 100

6. In standard decimal notation, how many positive integers less than 10,000 have at least one 3 as a digit?
- a. 3437 b. 3438 c. 3439 d. 3440 e. 3441
7. Ten writers covering a basketball league vote for the MVP of the league by listing their top three choices. A first place vote earns 5 points, a second place vote earns 3 points, and a third place vote earns one point. Manic Mary Monroe received 37 points. How many writers listed her on their ballots?
- a. 7 b. 8 c. 9 d. 10 e. cannot be determined
8. Three people are chosen at random from a large population. Assuming that it is equally likely to be born on a given day of the week, what is the probability that they were all born on different days of the week?
- a. $\frac{3}{7}$ b. $\frac{27}{49}$ c. $\frac{24}{49}$ d. $\frac{200}{343}$ e. $\frac{30}{49}$
9. The points $(6,4)$ and $(2,10)$ are symmetric with respect to the line L . Which of the following is an equation for line L ?
- a. $2x - 3y = 13$ b. $2y - 3x = 2$ c. $2x + 3y = 29$ d. $3x + 2y = 26$ e. $3y - 2x = 13$
10. Let $\sin\theta\cos\theta = \frac{7}{18}$. Compute $|\sin\theta + \cos\theta|$.
- a. $\frac{9}{7}$ b. $\frac{4}{3}$ c. 1 d. $\sqrt{2}$ e. $\frac{16}{9}$
11. What is the shortest distance between a point on the circle of radius 2 with center at $(0,4)$ and a point on the circle of radius 2 with center at $(4,0)$?
- a. $2 - \sqrt{2}$ b. $4 - \sqrt{2}$ c. $2\sqrt{2} - 2$ d. $4\sqrt{2} - 4$ e. 0

12. The determinant of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ x & 5x & 0 \\ 4 & x & 1 \end{bmatrix}$ is 100. What is the positive difference of the possible values of x ?

- a. 5 b. 0 c. 25 d. 10 e. 15

13. Two people agree to meet between 6 pm and 7 pm. They also agree to wait only 20 minutes for the other person to arrive. Assuming that arrival times are uniformly distributed over the one hour interval, what is the probability that they meet each other?

- a. $\frac{5}{9}$ b. $\frac{1}{3}$ c. $\frac{4}{9}$ d. $\frac{2}{5}$ e. $\frac{7}{16}$

14. The shadow of the school flag pole is 42 feet long. How tall is the flagpole if a meterstick next to the flagpole casts a 120 centimeter shadow?

- a. 45 ft b. 39 ft c. 30 ft d. 36 ft e. 35 ft

15. The sum of all integers from 1 to n , inclusive, is the same of the sum of all integers from n to 49, inclusive. What is the value of n ?

- a. 32 b. 35 c. 40 d. 38 e. 34

16. How many times in a 24-hour day are the minute hand and the hour hand (of a standard analog clock) perpendicular to each other?

- a. 42 b. 44 c. 46 d. 48 e. 49

17. Compute $\sum_{n=1}^{\infty} \frac{8}{3^n}$.

- a. 12 b. 10 c. 8 d. 4 e. 2

18. An equilateral triangle is inscribed in a circle of radius 6. A second equilateral triangle is circumscribed about the circle. If the sides of the triangles are parallel, what is the shortest distance from a point on one triangle to a point on the other?

- a. $\frac{3\sqrt{3}}{2}$ b. 4 c. $3\sqrt{3}$ d. $2\sqrt{3}$ e. 3

19. Sam and Bert play the following game: a fair coin is flipped repeatedly. Sam chooses a 3-outcome sequence, and then Bert chooses a different 3-outcome sequence. The winner is the sequence that occurs first. If Sam chooses HHH, which choice gives Bert the greatest probability of winning?

- a. THH b. THT c. TTH d. HTT e. TTT

20. Which of the following is the identity function $f(x) = x$ for all real numbers?

- a. $f(x) = \ln(e^x)$ b. $f(x) = e^{\ln(x)}$ c. $f(x) = \sqrt{x^2}$ d. $f(x) = \sin(\arcsin(x))$ e. all of these

21. Let $f(x) = ax + b$, with $b < a$ and both a and b positive integers. If for positive integers p and q , $f(p) = 18$ and $f(q) = 39$, what is the value of b ?

- a. 1 b. 3 c. 4 d. 5 e. 6

22. A circular table is pushed into a corner of a rectangular room so that it touches both walls. A point on the edge of the table and between the two points of contact is 2 inches from one wall and 9 inches from the other wall. Which of the following could be the radius of the table?
- a. 10 inches b. 12 inches c. 15 inches d. 17 inches e. 20 inches
23. In $\triangle SBC$, $SB = 12$, $BC = 15$, and $SC = 18$. Let O be the point for which BO bisects angle SBC and CO bisects angle SCB . If M and L are on sides SB and SC , respectively, such that ML is parallel to side BC and contains point O , what is the perimeter of $\triangle SML$?
- a. 24 b. 27 c. 30 d. 32 e. 36
24. The sum of two numbers is 1 and the sum of their squares is 2. What is the sum of their cubes?
- a. Cannot be determined b. 1 c. 1.5 d. 2.5 e. 5
25. Three friends joined a weight loss program. The oldest lost 10 pounds more than Kathy. The combined weight loss of Shirley and the youngest was 20 pounds more than Tim's loss. If Shirley is not the oldest, how many pounds did she lose?
- a. 20 b. 30 c. 0 d. 10 e. 25

SHORT ANSWER

Place the answer in the appropriate space.

66. Determine the value of k for which $\left|k - \left\|x\right\| - 6\right\| = 2$ has exactly 5 solutions.
67. An ant leaves his anthill for its morning walk. It walks 5 feet east and then makes a 160° turn to the right and walks 5 more feet. It then makes another 160° turn to the right and walks 5 more feet. If the ant continues this pattern until it reaches the anthill again, how far will it have walked?
68. You have a sheet of paper and a pair of scissors. Turn 1: You cut the paper into either 8 pieces or 12 pieces. Turn 2: You take one of these pieces and cut it into 8 pieces or 12 pieces. Hence after two turns you have 15, 19 or 23 pieces of paper sitting in front of you. There is no way to end up with 20 pieces of paper in front of you. However, if you continue the process eventually you can get all positive integers bigger than a particular integer. What is that largest integer that you can NOT obtain after several turns?
69. In parallelogram $ABCD$, \overline{BC} is extended beyond point C to point E . Points F and G are the points of intersection of \overline{AE} with \overline{BD} and \overline{CD} , respectively. If $FG = 12$ and $EG = 15$, determine AF .
70. Determine the smallest integer N such that N divided by n leaves a remainder of $n - 1$ for all integers $2 \leq n \leq 10$.

Answer Key

1. b
2. a
3. e
4. d
5. a
6. c
7. c
8. e
9. e
10. b
11. d
12. c
13. a
14. e
15. b
16. b
17. d
18. e
19. a
20. a
21. c
22. d
23. c
24. d
25. b

66. 8
67. 45 ft
68. 60
69. 18
70. 2519