Mesh Analyses and Simultaneous Equations

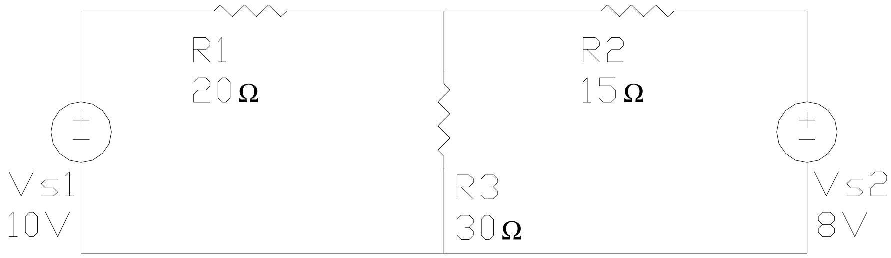


Figure 1

For the circuit in Figure 1, we wish to solve for the voltage and current across/through each of the resistors. To use Mesh Analyses, we must first draw a current in each loop of the circuit and indicate the voltage drops across each resistor as seen by each of these currents. This is shown below in Figure 2.

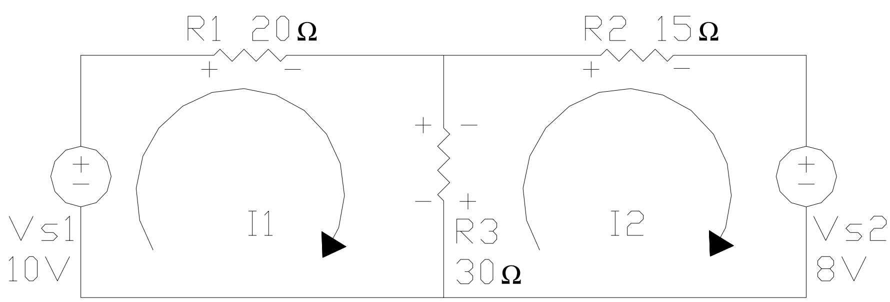


Figure 2

Here, both currents have been shown clockwise, but it does not matter which direction you choose as long as you mark the (+) to (-) polarities across the resistors in the direction of your current flow.

To solve this circuit, perform a KVL analysis on each of the currents of Figure 2.

Current I1:

Vs1 - I1*R1 - I1*R3 + I2*R3 = 0	(Eq. 1a)
I1(-R1 - R3) + I2(R3) = -Vs1	(Eq. 1b)
I1(-50) + I2(30) = -10	**(Eq. 1c)**

Current I2:

-I2*R3 + I1*R3 - I2*R2 - Vs2 = 0	(Eq. 2a)
I2(-R3 - R2) + I1(R3) = Vs2	(Eq. 2b)
I1(R3) + I2(-R3 - R2) = Vs2	(Eq. 2c)
I1(30) + I2(-45) = 8	**(Eq. 2d)**

Before proceeding, let's look at the equations for I1 and I2. For equations 1a and 2a, notice the voltage rises are positive and voltage drops are negative. As with the arbitrary current direction, these are also arbitrary choices--if you wanted, rises could be negative and drops positive. However, once you choose a system, make sure you are consistent. If you choose positive rises and negative drops for the first loop, use that system for the entire problem.

Next, notice that I1 and I2 share R3, but are moving in opposite directions through R3. In equation 1a, I1*R3 is a drop (and negative) since I1 is traveling from (+) to (-) through R3. I2, as seen by I1, is a rise (and positive) since I1 sees I2 moving up through the resistor. Another way of looking at this is that since I1 and I2 have opposite directions, they will also have opposite signs. Since I1 is a negative drop, I2 will be a positive rise as seen by I1.

In equation 2a, our point of reference changes--we must now examine how I2 sees the circuit. In the direction chosen for I2, R3, R2, and Vs2 are all seen as drops, so are negative. R3 is still sharing I1 and I2 in opposite directions. Since I2*R3 is negative in equation 2a, I1*R3 must be opposite, or positive.

Lastly, notice the change between equations 2b and 2c. The equations are the same, but the sequence of I1 and I2 have been swapped so the I1 and I2 are in the same positions relative to equation 1c. Although this step isn't necessary, doing it will reduce the chances of making a mistake later.

Now we have two equations and two unknowns:

I1(-50) + I2(30) = -10 **(Eq. 1c)**
I1(30) + I2(-45) = 8 **(Eq. 2d)**

Depending on your calculator, these equations can be solved in several different methods as shown below.

<u>Manual Method:</u>

Solving simultaneous equations manually requires eliminating one of the unknowns, solving for the other, and then finding the first one.

To solve one of the unknowns, multiply one equation through by some number so that the coefficients of one of the unknowns are equal in both equations. In this case, we can eliminate I2 by multiplying equation 1c by 1.5. This results in:

I1(-75) + I2(45) = -15 (Eq. 1c * 1.5)
I1(30) + I2(-45) = 8 (Eq. 2d)

These two equations can then be added together resulting in:

I1(-45) + I2(0) = -7

Solving this equation finds I1 = 155.56mA.

Plugging 155.56mA back into either 1c or 2d results in I2 = -74.07mA. The negative sign is telling us that I2 is really flowing counter-clockwise.

The current flow through R3 is found by I_{R3} = I1 - I2, if we assume the current is flowing down, or I_{R3} = I2 - I1 if we assume the current is flowing upwards. Since we know from our solution to I1 and I2 that I_{R3} is flowing down, I_{R3} = 155.56mA - (-74.07mA) = 229.6mA.

The voltage across each resistor can be found by Ohm's Law:

V_{R1} = I1 * R1 = 3.11V
V_{R2} = I2 * R2 = 1.11V (measured right-to-left for the correct current flow)
V_{R3} = I3 * R3 = 6.88V (measured top-to-bottom)

TI-83 Method:

TI-83 users will have to follow the Manual Method outlined above. TI-83+ users can download a flash application from TI that lets them use a method similar to the TI-85/86, below. This will require TI's Graphlink cable and software to transfer the application to the calculator. These are available in the ILC's Math Center. TI-83 Silver Edition users should have an application pre-loaded on their calculator that is also similar to the TI-85/86 method below.

TI-85/TI-86 Method:

The TI-85 and TI-86 have a built-in simultaneous-equation solver. One the TI-85, press <2nd><Stat>. On the TI-86, press <2nd><Table>. The resulting screen is shown below in Figure 3. Enter 2 for the number of variables and push <Enter> to proceed to the next screen as shown in Figure 4.

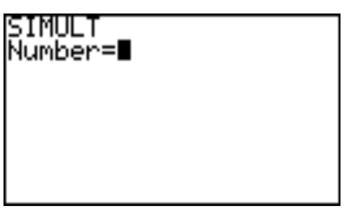


Figure 3

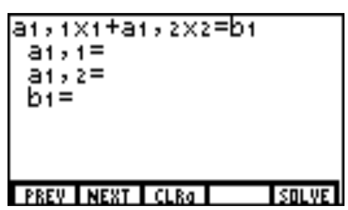


Figure 4

On this screen, $a_{1,1}$ corresponds to the -50 from equation 1c; $a_{1,2}$ is 30 and b_1 is -10. Enter these numbers, pushing <Enter> or the <down> key to proceed to the next line as shown in Figure 5.

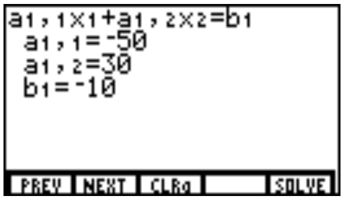


Figure 5

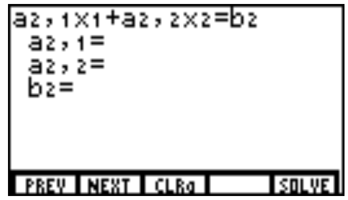


Figure 6

Once you have entered b_1, push <Enter>, <Down> or <F2> to go to the next screen as shown in Figure 6. On this screen, $a_{2,1}$ corresponds to 30 from equation 2d; $a_{2,2}$ is -45; b_2 is 8. Enter these numbers as you did for the first equation and push <F5> to solve. The resulting screen is shown Figure 7.

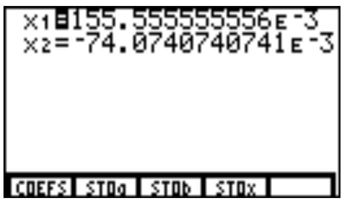


Figure 7

Figure 7 shows that I1 is 155mA and I2 is -74.07mA. You can check your coefficients or enter a new set of coefficients by pushing <F1> or push <Exit> to return to the home screen.

Note: you cannot save the values of x1 or x2 to memory directly, but using <F4>, you can store x1 and x2 as a matrix. Likewise, the "a" and "b" values cannot be stored directly but can be saved as matrices using <F2> and <F3> while in the screen of Figure 7.

The TI-85/86 does this because as you are entering the values for Figuers 5 and 6, you are actually filling in the values of two matrices as illustrated below:

Equations:

$$I1(-50) + I2(30) = -10$$
$$I1(30) + I2(-45) = 8$$

Matrix:

$$\begin{vmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{vmatrix} \begin{vmatrix} I1 \\ I2 \end{vmatrix} = \begin{vmatrix} b_1 \\ b_2 \end{vmatrix}$$

$$\begin{vmatrix} -50 & 30 \\ 30 & -45 \end{vmatrix} \begin{vmatrix} x1 \\ x2 \end{vmatrix} = \begin{vmatrix} -10 \\ 8 \end{vmatrix}$$

When you solve for I1 (x1) and I2 (x2), the TI-85/86 is actually performing matrix operations to solve the 2-row by 1-column matrix of x1 and x2.

TI-89 Method:

The two equations we derived for the original problem are

I1(-50) + I2(30) = -10 (Eq. 1c)
I1(30) + I2(-45) = 8 (Eq. 2d)

Like the TI-86, the TI-89 uses matrices to solve I1 and I2. One matrix (2x2) is required for the coefficients on the left side of the equation, and one matrix (2x1) is required for the coefficients on the right side of the equation. Unlike the TI-86, the TI-89 does not have a GUI screen for simultaneous equations. This means that you must manually enter these matrices.

Matrices on the TI-89 are entered using the brackets, commas, and semi-colons. The matrices

$$\begin{vmatrix} -50 & 30 \\ 30 & -45 \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} -10 \\ 8 \end{vmatrix}$$

would be entered on the TI-89 as

[-50,30;30,-45] and [-10;8]

The brackets define the matrix, the commas separate row items, and the semi-colons separate the rows.

To solve the simultaneous equations presented, you must use the **simult()** function in the TI-89 catalog. To access this function, push the <Catalog> key and then push the <3> key, which will jump you directly to the "S" items, and then scroll down to "simult(".

The correct item is shown in Figure 8. Push <Enter> to select "simult(" as shown in Figure 9.

Figure 8

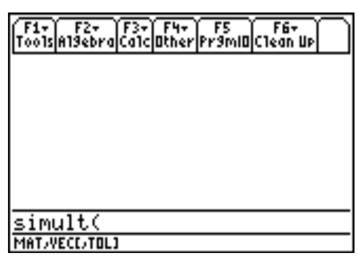


Figure 9

Next, enter the matrices separated by a comma, and close the parentheses. A portion of the line is shown in Figure 10.

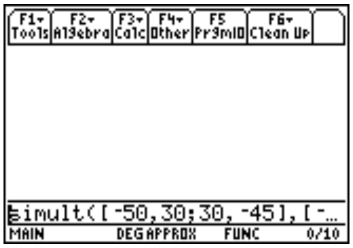


Figure 10

The entire line of text for Figure 10 should appear as:

simult([-50,30;30,-45],[-10;8])

Push <Enter> and the results are shown in a matrix as in Figure 11 below:

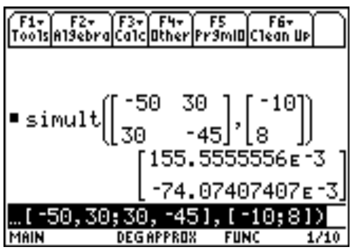


Figure 11

This matrix confirms I1=155.56mA and I2= -74.07mA.

If you desire, you can save any of these matrices into memory and/or solve them by their memory names. Figures 12 and 13 show saving the two known matrices into variables "a" and "b". The → symbol appears when you press the <sto→> key near the bottom left of the keypad. For example, the keystrokes for Figure 12 are

[-50,30;30,-45] <sto→> <alpha> <=>

Note that the <=> key for the "equal sign" is **NOT** the same as the <Enter> key. The <=> key is on the middle-left of the keypad, underneath the letter "A".

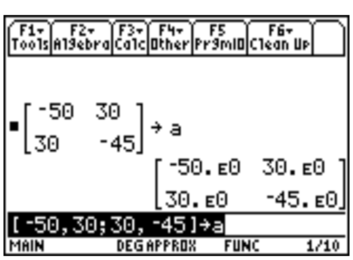


Figure 12

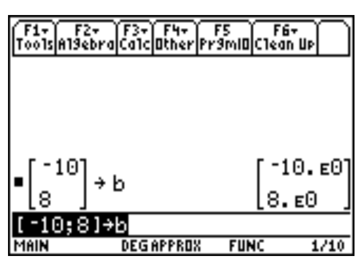
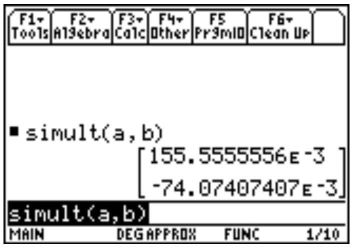


Figure 13

Figure 14 then shows using the simult() function to solve the matrices using their memory names. Figure 15 shows saving the resulting matrix into memory.



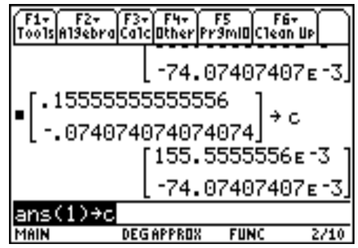


Figure 14

Figure 15

Matrices for the TI-89 can also be entered and saved into memory using the Matrix Editor. Consult your TI-89 User’s Manual on how to access and use the Matrix Editor.

As with the TI-86, there are programs and flash applications available from the TI and various other websites that will give the TI-89 a GUI interface like the TI-86 for simultaneous equations if you prefer to use one.

The website to download some of the flash applications available for TI calculators is:

http://education.ti.com/us/product/prselect.html