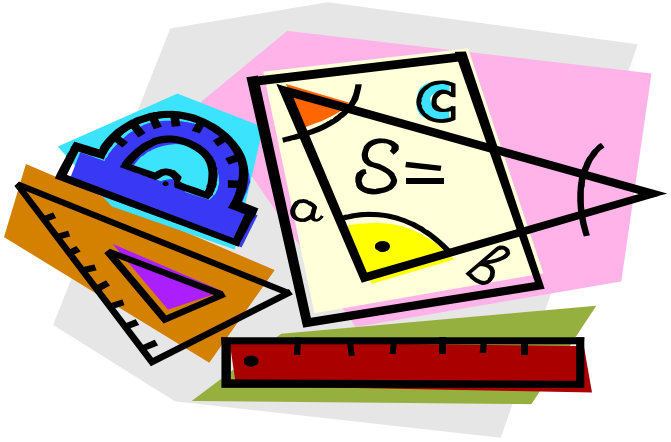
Geometry



Do NOT open until you are told to do so.

March 25, 2010

1. The tallest man-made structure in the world is the Burj Khalifa in Dubai, UAE, at 828 meters high. How far away is Bill standing from the center of the Burj Khalifa to the nearest meter if his angle of elevation to the top is 30°?

   a. 1434 m b. 1656 m c. 2868 m d. 1528 m e. 1559 m

2. A square is inscribed in a circle of radius 10 inches. What is the area of the square?

   a. 100 sq in b. 200 sq in c. 50π sq in d. 100π sq in e. 400 sq in

3. Given two concentric circles of radii 12 inches and 4 inches, find the length of a chord to the nearest inch in the larger circle that is tangent to the smaller circle.

   a. 11 in b. 22 in c. 12 in d. 23 in e. 24 in

4. Tony's Pizza Palace likes to cover its pizzas with 0.25 in^3 of sauce per square inch. A worker has a cylindrical container of sauce of height 8 inches and diameter 6 inches. How many 12 inch diameter pizzas can the worker cover with the sauce?

   a. 6 b. 8 c. 9 d. 10 e. 12

5. You are in a land inhabited by people who either always tell the truth or always tell falsehoods. You come to a fork in the road and you need to know which fork leads to the capitol. There is a local resident there, but he has time to only reply true or false to one statement. Which of the following statements should you make to discover the fork that leads to the capitol?

   Statement A: "The left fork leads to the capitol if and only if the right one does not."
   Statement B: "I am on the road to the capitol"
   Statement C: "I am lost if and only if this is the road to the capitol"
   Statement D: "You are an honest man if and only if the right fork leads to the capitol."
   Statement E: "If the left fork leads to the capitol, then you are a local resident."

   a. A b. B c. C d. D e. E

6. The manufacturers of a certain pen claim it can draw a line 1 kilometer long. If the line is 0.4 mm wide, how large is the area covered by the line in square meters?

a. 4000 sq m b. 400 sq m c. 40 sq m d. 4 sq m e. 0.4 sq m

7. On a baseball diamond, the distance between bases is 90 feet and the shortstop's position is exactly halfway between third and second base. How much longer is the throw from third to first base than the throw from shortstop to first base to the nearest foot?

a. 25 feet b. 27 feet c. 29 feet d. 31 feet e. 33 feet

8. Compute the volume of a rectangular prism that has faces with areas 72 cm^2, 120 cm^2, and 135 cm^2.

a. 1680 cm^3 b. 1008 cm^3 c. 1080 cm^3 d. 450 cm^3 e. 1260 cm^3

9. If a recipe for a cake to be baked in a rectangular pan calls for 3 eggs and the baker doubles each dimension of the pan, how many eggs will be required, assuming that the baker wants to fill the new pan?

a. 6 b. 8 c. 12 d. 24 e. 32

10. A ladder leans against a wall. The top of the ladder is 7 feet above the ground. If the ladder is then moved a foot farther from the wall, the ladder will lie on the ground with its top touching the wall. How long is the ladder to the nearest foot?

a. 25 ft b. 24 ft c. 16 ft d. 17 ft e. 20 ft

11. A *Norman window* has the shape of a rectangle surmounted by a semicircle. If the perimeter of a Norman window is 30 ft and the diameter of the semicircle is 6 ft, find the area to the nearest hundredth of a square foot.

a. 59.23 ft^2 b. 58.35 ft^2 c. 57.86 ft^2 d. 56.34 ft^2 e. 61.75 ft^2

12. A circle of radius 2 and center E is inscribed inside a square ABCD. Find the area that is inside ΔABE but outside the circle.

a. $\pi - 3$ b. $\frac{\pi}{2} - 1$ c. $4 - \pi$ d. $\pi - 2$ e. $3 - \frac{\pi}{2}$

13. Find the area of the convex polygon in the plane with vertices at the points whose coordinates are $(-2,3)$; $(1,10)$; $(5,10)$; $(8,7)$; and $(4,0)$.

a. 54 sq units b. 55 sq units c. 56 sq units d. 58 sq units e. 62 sq units

14. Point P in the interior of the triangle ABC is equidistant from the triangle's vertices. If $m\angle$ ABP = 40° and $m\angle$ CBP = 30°, then what is $m\angle$ PAC?

a. 20° b. 25° c. 30° d. 40° e. 45°

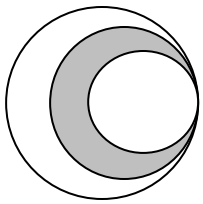
15. How many diagonals are there in a 17-sided convex polygon?

a. 136 b. 129 c. 119 d. 117 e. 153

16. A floor tile has the shape of a regular polygon. If the tile is removed from the floor and rotated through 50° it will fit back into its original place in the floor. What is the minimum number of sides of the regular polygon?

a. 8 b. 24 c. 25 d. 30 e. 36

17. Three circles A, B, and C have one point in common with circle A entirely inside circle B which is entirely inside circle C as shown in the figure. The radius of circle A is two-thirds the radius of circle B and the radius of circle B is three-fourths the radius of circle C. What part of the area of circle C is the shaded region (the area inside circle B and outside circle A)?



a. $\frac{1}{12}$ b. $\frac{17}{144}$ c. $\frac{1}{4}$ d. $\frac{5}{16}$ e. $\frac{9}{16}$

18. In triangle PQR, $m\angle$ PQR = 30°, $m\angle$ PRQ = 105°, and side QR has length 8 inches. What is the area of triangle PQR to the nearest hundredth of a square inch?

a. 24.95 in² b. 23.94 in² c. 22.95 in² d. 21.43 in² e. 21.86 in²

19. If x is the perimeter of a regular hexagon circumscribed about a circle and y is the perimeter of a regular hexagon inscribed in the same circle, then what is the ratio of x to y?

a. $\frac{2\sqrt{2}}{3}$ b. $\frac{2\sqrt{3}}{3}$ c. $\sqrt{2}$ d. $\frac{\sqrt{3}}{2}$ e. $\frac{\sqrt{3}}{3}$

20. A circular piece of paper with radius 5 cm has a sector with an angle of 144° removed. The radius edges of the remaining sector are joined to make a right circular cone whose tip is the center of the circle and whose slant height is 5 cm. What is the volume of the cone in cubic centimeters?

a. 4π cm³ b. 6π cm³ c. 8π cm³ d. 12π cm³ e. 18π cm³

21. Jill's bicycle wheels have a diameter of 27 inches. She rides her bicycle to school which is 4 miles from home. Which of the numbers most closely approximates the number of revolutions made by one wheel during the trip from home to school?

a. 1000 b. 2000 c. 3000 d. 4000 e. 5000

22. A rectangular tank with a square base of 5 meters contains water to a depth of 4 meters. A solid cube of edge 3 meters is placed at the bottom of the tank. What is the new level of the water in the tank?

a. 5.4 m b. 57 m c. 5.08 m d. 6.65 m e. 5.02 m

23. Between 10:52 am and 3:18 pm on a given day, the minute hand of a standard wall clock sweeps out an area of 28 cm². What was the distance to the nearest tenth of a centimeter traveled by the tip of the minute hand?

a. 39.5 cm b. 38.2 cm c. 41.3 cm d. 38.4 cm e. 41.5 cm

24. In the 1960s every playground in the country had a metal slide that was 10 feet tall and very steep. In the late 1970s federal regulations put an end to tall, steep metal slides. The new regulations require that the length of a slide be more than or equal to twice its height. Assuming the end of the slide is on the ground, what is the tallest a slide can be to the nearest hundredth of a foot if the horizontal distance from the end of the slide to the base of the ladder is 6.5 feet?

a. 4.5 ft b. 3.85 ft c. 3.45 ft d. 3.75 ft e. 4.25 ft

25. Recall the following Roman Numeral values: VII = 7: IX = 9; XXV = 25; XLVI = 46; XCII = 92; and MDCLXI = 1661. Compute the volume of a rectangular prism of sides XVII, XV, and XII.

a. MMLX b. MMDXC c. MMMXC d. MMDCL e. MMMLX

SHORT ANSWER
Place the answer in the appropriate space.

66. Let A, B, C, D, and E represent the measures of the five interior angles of a convex pentagon. If the sum of A, B, and C is 345° and D is the average of A, B, C, and E; what is the value of E?

67. A woman 5ft 3 in tall walked around the earth at the equator. The path traced by the top of her head is how much longer to the nearest foot than the path traced by the soles of her feet?

68. Suppose the mosquito population of the world is 36 trillion and that those mosquitoes can be packed into a cubic box with no space wasted. If the average mosquito is 6 mm^3, how long would the edge of the box have to be in meters?

69. A triangle has sides of length 11, 13, and 16 inches. How long is the median to the side of length 16 inches?

70. The perimeter of a semicircle region is 50 yards. Find the area to the nearest tenth of a square yard.

1. a
2. b
3. d
4. b
5. d
6. e
7. b
8. c
9. d
10. a
11. c
12. c
13. e
14. a
15. c
16. e
17. d
18. e
19. b
20. d
21. c
22. c
23. a
24. d
25. e

66. 87°
67. 33 ft
68. 60 m
69. 9 in
70. 148.5 yd^2

1. $\sqrt{\left(1656^2 - 828^2\right)} \approx 1434$ m.

2. $\frac{20 * 20}{2} = 200$ sq in.

3. $2\sqrt{\left(12^2 - 4^2\right)} \approx 23$ in.

4. The area of a 12 in pizza is 36π square inches so one pizza requires 9π in³ of sauce. The can of sauce contains 72π in³ of sauce. That means it will cover exactly 8 pizzas.

5. Statements A and E are always true so the local resident's response will reveal nothing except whether the local resident always tells the truth or always tells falsehoods. Statements B and C do not mention which fork to take so again no help. Statement D is true if the right fork leads to the capitol and the person is an honest man. It is also true when the left fork leads to the capitol and the person tells falsehoods. So no matter who is responding if the answer is "true" you take the right fork and if the answer is "false" you take the left fork.

6. 1 kilometer = 1000 meters, 0.4 millimeters = 0.0004 meters. So (1000 meters)(0.0004 meters) = 0.4 m².

7. $\sqrt{\left(90^2 + 90^2\right)} - \sqrt{\left(90^2 + 45^2\right)} \approx 27$ ft.

8. $\sqrt{(72 * 120 * 135)} = 1080$ cm²

9. Doubling each dimension increases the volume by 8 so 8*3 = 24 eggs will be required.

10. Let x be the original distance of the ladder from the wall. Then the length of the ladder is $x + 1$. $(x+1)^2 = x^2 + 49 \Rightarrow x^2 + 2x + 1 = x^2 + 49 \Rightarrow 2x = 48 \Rightarrow x = 24$. Hence the ladder is 25 feet.

11. Let x be one side of the height of the rectangle, then $2x + 6 + 3\pi = 30 \Rightarrow x = 12 - 1.5\pi$. Hence the area of the window is $6(12 - 1.5\pi) + 4.5\pi \approx 57.86$ ft².

12. Triangle ABE has area 4. The sector of the circle in triangle ABE has area $\frac{4\pi}{4} = \pi$. Hence the area in question is $4 - \pi$.

13. There are several ways to compute the area of the pentagon in the coordinate plane, but the easiest way is to draw the rectangle with coordinates $(-2,0)$; $(-2,10)$; $(8,10)$; and $(8,0)$ which has area 100. Then subtract the triangles at each corner $100 - \frac{4(7)}{2} - \frac{3(3)}{2} - \frac{3(7)}{2} - \frac{3(6)}{2} = 62$ sq unit

14. ΔAPB, ΔBPC, and ΔCPA are all isosceles triangles. Thus, $m\angle PAC = \frac{180 - 30 - 30 - 40 - 40}{2} = 20°$.

15. The number of diagonals in a convex polygon is the number of vertices times the number of vertices less three divided by two. If there are 17 vertices there are 119 diagonals.

16. For the tile to fit back in 50° must be divisible by the degree of the central angle. The degree of the central angle of an *n*-sided regular polygon is $\frac{360}{n}$. The smallest number that gives a number that evenly divides 50 is 36.

17. Let *r* be the radius of circle H, then the radius of circle G is 1.5*r* and the radius of circle F is 2*r*. Hence the ratio of the shaded area to the area of circle F is $\frac{\left(1.5^2-1^2\right)r\pi}{4r\pi}=\frac{1.25}{4}=\frac{5}{16}$.

18. Draw the altitude to side PQ and let the intersection be at point T. Then ΔRTQ is a 30-60-90 right triangle with hypotenuse 8 inches and ΔPRT a 45-45-90 right triangle. This means RT = PT = 4 inches and
TQ = $4\sqrt{3}$ inches. Thus the area of triangle PQR is 21.86 in².

19. Let *r* be the radius of the circle. The perimeter of the inscribed hexagon is 6*r* and the perimeter of the circumscribed circle is $4r\sqrt{3}$. So the ratio is $\frac{2\sqrt{3}}{3}$.

20. The sector remaining has a central angle of 216°. The circumference of the base of the cone is then $\frac{216(10\pi)}{360}=6\pi$ cm. This means the radius of the base is 3 cm. Since the slant height is 5 cm that means the height of the cone is 4 cm. Hence the area is $\frac{1}{3}(9\pi)(4)=12\pi$ cm³.

21. The circumference of Jill's bike tire is 27π inches or $\frac{9\pi}{4}$ feet which is $\frac{9\pi}{4(5280)}$ miles. Hence her tire makes $4\div\frac{9\pi}{4(5280)}=\frac{16(5280)}{9\pi}\approx 2988$ which is closest to 3000 revolutions.

22. The volume of the original water in the tank is 100 m³ and the volume of the solid cube is 27 m³ so the new volume must be 127 m³. Hence the new height must be $\frac{127}{25}=5.08$ m.

23. The minute hand travels around the clock $\frac{133}{30}$ times. The area of the circle traveled by the minute hand is $28\Big/\left(133\big/30\right)=\frac{120}{19}$ cm². This means the circumference of the circle traveled by the minute hand is $2\pi\sqrt{\left(\frac{120}{19\pi}\right)}$ cm. Thus the total distance is $2\pi\sqrt{\left(\frac{120}{19\pi}\right)\left(\frac{133}{30}\right)}\approx 39.5$ cm.

24. The new federal regulations translate into the base angle being less than or equal to 30°. Thus the tallest the slide can be is $\frac{6.5}{\sqrt{3}} \approx 3.75$ feet

25. XVII = 17; XV = 15; and XII = 12. Hence the volume is 3060 or MMMLX.

66. The sum of the degrees of a convex pentagon is 540°. This means that $D = \frac{345 + E}{4}$ and $D = 195 - E$. Hence E is 87°.

67. Let *r* be the radius of the earth, then $2(r + 5.25)\pi - 2r\pi = 10.5\pi \approx 33$ ft

68. $\sqrt[3]{((36{,}000{,}000{,}000)6)} = 60{,}000$ mm or 60 m.

69. Let *x* be the length of the median, then by Heron's Theorem the area of the triangle is $\sqrt{20(9)(7)(4)} = 6\sqrt{140}$. So the area of the triangle with sides 11, 8, and *x* must be $3\sqrt{140}$ and applying Heron's Theorem again gives the equation $\left(\frac{19+x}{2}\right)\left(\frac{19-x}{2}\right)\left(\frac{x+3}{2}\right)\left(\frac{x-3}{2}\right) = 1260$. Hence $(361 - x^2)(x^2 - 9) = 20160 \Rightarrow -x^4 + 370x^2 - 3249 = 20160 \Rightarrow x^4 - 370x^2 - 23409 = 0$. Factoring gives $(x^2 - 81)(x^2 - 289) = 0$. This gives solutions 9 and 17. The 17 is too big so the median is 9 in.

70. Let *r* be the radius of the region then $2r + r\pi = 50 \Rightarrow r = \frac{50}{2 + \pi}$. Hence the area is $\left(\left(\frac{50}{2+\pi}\right)^2 \pi\right) \Big/ 2 \approx 148.5$ yd².