Connecting Industry to Mathematics Instruction
NSF ATE Award # 1954291

Long and Winding Road - Teacher Notes

A Practice Understanding Task

In partnership with
WAKE TECH
WAKE COUNTY PUBLIC SCHOOL SYSTEM
WakeEd PARTNERSHIP

Purpose: In this activity, students will examine how curves in roads are designed and how speed limits are set based on the amount of curvature and banking that exists in the design.
Students will also look at redesigning an existing road for safe travel at a high speed.

Career Field: Civil Engineering and Geomatics, NC DOT

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NC Math 4 Standards:

NC.M4. AF.2.2 Implement the Law of Sines and the Law of Cosines to solve problems.

NC.M4.AF.5 Understand how to model functions with regression.

NC.M4.AF.5.1 Construct regression models of linear, quadratic, exponential, logarithmic, & sinusoidal functions of bivariate data using technology to model data and solve problems.

Unit Alignment:

Indicate where this lesson would be used in the course

NC Math 4 - Unit (Law of Sines and Cosines / Quad Regressions)
WTCC Math 121

Common Core State Standards for Mathematical Practice

Indicate which of the standards are highlighted in this lesson

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. Construct viable arguments and critique the reasoning of others.
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. Look for and express regularity in repeated reasoning.

Prerequisite Skills

List any prerequisite skills that may need to be addressed in a warm-up

- Find slope of incline given an angle of incline
- Use the distance formula to find distance between two given points (x,y)
- Use Law of Cosines/Sines to find the angles given all 3 sides of a triangle
- Regression modeling and predicting with Quadratics

Time Required

The time required to complete this activity is approximately 120 minutes.
Task 1 is an introduction into superelevation being the slope of a road banked from side to side. Task2 builds on superelevation understanding and computing regression to relate the bending of a curve (radius of circular arc) and the speed limit of the curve for safe travel. Task 3 involves determining the radius of a curving road only given 3 points along the road design path. Task 3, parts F and G are extensions and can be left off the project if time is an issue. They involve redesigning the road to be traveled at 65 mph and also finding a cost estimate of the redesign. A suggested strategy is to do task 1 for the last 15 minutes of a class day, then complete task 2 and task 3 in the next class day. Then challenge students to complete task 3F and G overnight and have the discussion phase at the beginning of the third class. This gives students the opportunity to reflect on their work, making the task more meaningful and long-lasting.

Materials Needed

- Students Activity Sheet
- Spreadsheet technology or Desmos to compute regression given data

Vocabulary

- e% is the slope written as a percent – 4% equates to a slope of 4/100 (rise over run)
- Superelevation of a road – side to side tilt or banking of the road that helps safe travel along a sharp curve and it also helps to drain water off the road in a downpour
- Horizontal Curve – NC DOT uses circular arcs to model the left or right bending of a road
- Vertical Curve – NC DOT use parabolas to model the up and down travel along a road

The Teaching Cycle:

Maybe the Desmos Launch could be done for homework the night before you plan to start the activity.
Launch: Ask students to consider a car racetrack and discuss why the slope of the curves are so steep. What could happen if the slope of the bank is less, and the curves is a sharper bend? Get at the idea that a relationship exist between the superelevation, the sharpness of the bending curve, and the maximum speed for safe travel.

Show the Launch video.
Invite questions about ideas or terms encountered in the video or in the introductory paragraphs of the Student Activity sheet.

Explore Task1: This is an introductory task about super-elevation of a road. After watching the launch video, it should be an easy set of tasks that is great done in groups. Several discussion questions exist (with no math) that get the student thinking about the relations between curve amounts, superelevation, and speed limits. In the field, slope is expressed as a percent. For example, 5% slope means 5/100, or in other words for every 100 feet in horizontal distance, the road will drop/rise 5 feet vertically. The only problem area for students here is finding slope from a given angle of incline in the banking of a road. You may want to hint to students that they can draw a triangle with the hypotenuse along the road surface and the legs will be the rise and run of the slope. They should use tangent right triangle trig to solve for the slope. Writing the slope as a fraction with 100 in the denominator will then represent the % of the superelevation in the banking of the road in the curve.

Explore Task2: Students are exploring the relationship between the superelevation, radius of curvature, and the speed limit in this set of tasks. They start with a table of values that show some numerical corresponding values between the three quantities. The first task is just asking the students to explain what a couple of values mean. They should notice that if you increase the superelevation and decrease the radius of the curve (which will sharpen the amount of bending in the road), the speed limit can actually roughly stay the same. In other words, more banking in the road and making the road bend sharper can offset each other allowing the speed limit to stay basically the same. This task ends with mathematically finding the quadratic equation that relates the speed and radius of curvature in the road for a given percent of superelevation. Students will then use the regression equation to predict some values.

Excel or Desmos are great tools to perform the regression equations for this task. However, Excel is the tool mostly used in Industry, so the more students use this tool, the better equipped they will be to enter the workforce with important skills they will use in their job.

Explore Task3: Students will be challenged in this set of tasks. You may want to pre-load them with the idea of a Station. Surveyors take readings from a Station, which is just a point that measurements are taken from. The data provided for points along a curved road is taken from a Station, and students will then adjust the points to (x,y) Cartesian coordinates. North direction will become the y-coordinate and the East direction will become the x-coordinate. Students then know the 3 corner points of a triangle and using the distance formula (also reviewed in the Desmos Launch teacher activity), students can find the lengths of the 3 sides and therefore find the interior angles using the Law of Cosines and Sines. Finding the radius of the curve will require the students to "look" through the set of given formulas to find the one that will output the radius knowing parts of the triangle that they solved. One word of caution: when we teach Law of Sines and Cosines, A, B, C are always opposite sides a, b, c. The formulas used by NC DOT do NOT use this schematic. Students need to relate the letters in the formula to the drawing given with the formulas. Knowing the radius and superelevation in the curve will allow them to find the safe speed for the road. This uses the information from Task 2. Parts F and G are extensions and could be left off the assignment if time is an issue. Part F asks the students to redesign the road by moving the middle point B in order to keep the speed limit 65 mph. If Excel is used for regression, Goal Seek in What-If-Analysis is a great tool to predict the input given the output value of the function. This question is basically computing the last several questions in reverse. This question is the most challenging question in the activity. Then, part G is computing the projected cost of the redesign to NC DOT. Students will use the arc length formula for a section of a circle ($s = \Theta \cdot R$).

Discussion: Students will share their results and compare what they arrived at compared to other groups. Groups can explain their process they used at arriving at their result. You could have groups trade their answers and have groups compare their results with the results of the paper they are reviewing. Again, time is a factor in how you handle the discussion part of the activity. Do at least have each group explain to some degree how they arrived at some part of their results.

OnLine Class Option: Below find the blackboard post for this activity assignment for an online class. The discussion part of this schematic could be assigned as the Discussion part of an in class face to face class.

Project Steps

Step 1: **GOAL COMPLETION BY Wednesday 4/20.**

- Complete the Launch Desmos Activity: "Long and Winding Road" Desmos Link: https://student.desmos.com/join/BDJ9D8
- Read through the activity and use your MS TEAMS groups to chat to begin to understand the goals of the project .
- You will have the same groups for this activity as you had for the second Industry Project.

Step 2: **GOAL COMPLETION BY FRIDAY 4/22. MUST BE COMPLETE BY MONDAY 4/25 @ 11:59 PM.**

- Explore the task as a group.
- This does not mean, divide up the tasks into individual pieces but as a group, discuss the problem and agree on the best solution to the problem.
- As part of the explore phase, your group will be expected to solve the problem and submit one copy of your answers as a group via blackboard.
- I strongly encourage you to set up at several teams meetings to work through this as a group. Please invite me to any group teams meetings you have. I will attend as many as I can.

Step 3: **MUST BE COMPLETE BY MONDAY 4/25 @ 11:59 PM.**

- There is a discuss portion of this task at the end.
- The discuss portion is important to this project because it allows you to see how other students approached their problems and whether or not you all had the same thought process.

- Your group will produce an electronic explanation of your approach and solutions to All questions in the project.
- You can choose from the following list for the discuss portion of the project:
  - A video
  - An MS Teams meeting recording (invite me to it and record it for posting on blackboard)

Exit Ticket: For a horizontal curve with superelevation of 2.4%, what safe speed limit should be set if the radius of the circular curve is 3500 feet? Or, for a horizontal curve or radius 2000 ft, what superelevation would be required to have a safe speed limit of 45 mph? These questions will require the students to estimate the answers from the table given in task 2 or they could do a regression to mathematically predict the answers.

This activity could be extended into a larger project and have a NC DOT employee in Road Design invited in to talk with students about their result and future processes that are being used in road design. Virtual engagements are also possible with the NC DOT Road Design Unit.