Connecting Industry to Mathematics Instruction
NSF ATE Award # 1954291

Student Activity Sheet
Long and Winding Road

In partnership with
WAKE TECH
WAKE COUNTY PUBLIC SCHOOL SYSTEM
WakeEd PARTNERSHIP

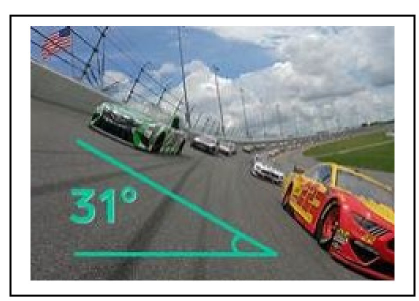
Speed limits on curved roads, often shown on cautioned signs, are set based on several factors to keep travel safe in many conditions. Many of the conditions were established following the construction of roads. First, you will explore the relationships that exist between the speed limits set along a curve and the factors that are considered in determining a safe speed when traveling along a curved road. Then you will use data determined by geomatics (surveying) along a curved road to determine if the existing speed limit set is safe enough for travel.

Task 1. Introductory thought and estimation tasks:

A) Name at least 3 factors that could be used to determine a safe speed to travel along a curved road.

B) For the two speed limit signs to the right, what is the main factor that determines one speed is lower than the other? Why?

C) For the racetrack picture shown to the right, how does the 31 degree slope affect the safe speed traveling along a curved road?



D) What is the slope of the racetrack surface shown in part C? Express the slope as a fraction with 100 as the denominator.

E) When the slope is written as a percent (5% = 5/100), it is called the **superelevation** of the road. Estimate the superelevation of the curved road below and explain your reasoning.

Task 2. The table below is used by NC DOT Road Design Engineers and it relates the safe speed, or velocity (V) of a curved road, the radius (R) of the circle that defines the path of the road, and the superelevation (e%) of the banked road. Use the table to explore the relationships that exist between these variables. NC is "normal crown" and RC is "remove adverse curve", but these are not used in this task.

U.S. Customary										
e (%)	V_d = 15 mph	V_d = 20 mph	V_d = 25 mph	V_d = 30 mph	V_d = 35 mph	V_d = 40 mph	V_d = 45 mph	V_d = 50 mph	V_d = 55 mph	V_d = 60 mph
	R (ft)	R (ft)	R (ft)	R (ft)	R (ft)	R (ft)	R (ft)	R (ft)	R (ft)	R (ft)
NC	796	1410	2050	2830	3730	4770	5930	7220	8650	10300
RC	506	902	1340	1880	2490	3220	4040	4940	5950	7080
2.2	399	723	1110	1580	2120	2760	3480	4280	5180	6190
2.4	271	513	838	1270	1760	2340	2980	3690	4500	5410
2.6	201	388	650	1000	1420	1930	2490	3130	3870	4700
2.8	157	308	524	817	1170	1620	2100	2660	3310	4060
3.0	127	251	433	681	982	1370	1800	2290	2860	3530
3.2	105	209	363	576	835	1180	1550	1980	2490	3090
3.4	88	175	307	490	714	1010	1340	1720	2170	2700
3.6	73	147	259	416	610	865	1150	1480	1880	2350
3.8	61	122	215	348	512	730	970	1260	1600	2010
4.0	42	86	154	250	371	533	711	926	1190	1500

A) In the above table, the values 838 and 835 appear. Explain what each appearance means including units on all your values and why two values so close can be in the same chart.

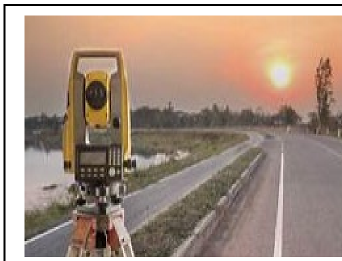
B) For a safe speed of 55 MPH, as the superelevation increases, how does the radius of the curved road change? Explain the contextual reason for this.

C) For a superelevation of 4%, as the radius of the curved road increases, how does the safe speed of the curved road change? Explain the contextual reason for this.

D) For a superelevation of 4%, find the mathematical relationship between the radius of the curved road and the safe speed for traveling the road. (regression)

E) Using your model from part D, predict the safe speed limit allowed for a Horizontal Curve of radius 2000 meters. (Careful with units)

Task 3. Now that you know the relationship between a curved road radius and the speed limit for safe travel, you are tasked with determining if an existing curved road is safe to travel under the current speed limit set. You will use data provided by surveyors that report to you three locations along the curved road. You need to determine the radius of the curved road from the three locations given. Then use it to determine if the current speed limit is acceptable for safe travel. A Geomatics graduate surveys the curve and reports the following three locations on the midline of the road in the curve.



Surveyors measure distances from a set location, called STATIONS. A station is chosen such that all possible designs will stay in Quadrant I. The current speed limit on the curved road is 65 MPH. Assume 4% superelevation for all the following tasks.

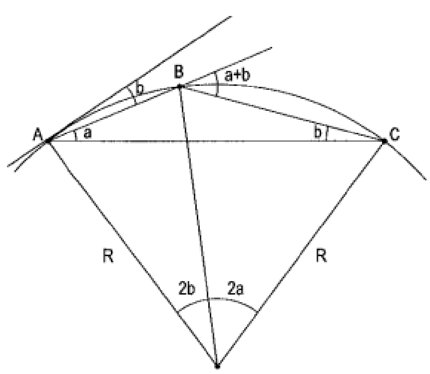
To reduce these numbers to make them more manageable, set a new STATION at 9000 feet North and 3000 feet East from the original STATION. Also, change the Northern distances to be the second number in your coordinate location and the Eastern distances to be the first number in your coordinate location.

A) Complete the table below for the adjusted locations of points A, B, and C using the STATION at 9000 feet North and 3000 feet as the location (0,0). (Remember: Change the adjusted Northern distance to the y-coordinate and change the adjusted Eastern distance to the x-coordinate.)

Point Labels	North from Station (ft)	East from Station (ft)
A	9249	3701
B	9196	3824
C	9161	3958

Point Labels	Horizontal x-axis	Vertical y-axis
A		
B		
C		

Road Design Engineers use circles to model curves in roads and they use the term HORIZONTAL CURVES. Below is a diagram of a horizontal curve and a common formula used to find the radius of the horizontal curve.

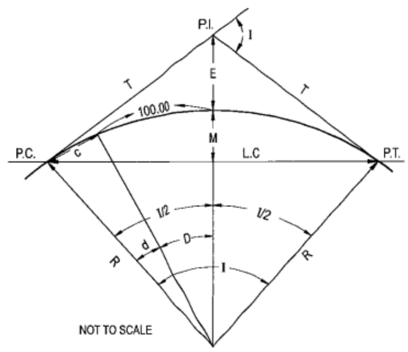


$$R = \frac{AC}{2\sin(a+b)}$$

HORIZONTAL CIRCULAR CURVES

D = Degree of curve, arc definition
L = Length of curve from P.C. to P.T.
c = Length of sub-chord
ℓ = Length of arc for sub-chord
d = Central angle for sub-chord

$$D = \frac{5{,}729.58}{R}$$
$$T = R\tan(I/2)$$
$$L = RI\frac{\pi}{180} = \frac{I}{D}(100)$$
$$LC = 2R\ \sin(I/2)$$
$$c = 2R\sin(d/2)$$
$$d = \ell D/100$$
$$M = R[1-\cos(I/2)]$$



B) Determine the distances between each of the points A, B, and C.

Segment	Length
AB	
BC	
AC	

C) Determine the interior angles of the triangle ABC.

Angle	Degrees
a	
b	

C	

D) Determine the radius of the curved road.

E) Is the current speed limit set on this curve safe to travel? Why or why not?

F) If not, adjust the location of point B, so that it is safe to travel at 65 MPH. Give the location of the new point B.

G) Construction costs for a two lane road is between $3 million per mile and $5 million per mile in urban areas. Assuming a cost of $4 million per mile, determine a projected cost of constructing the new designed curve.